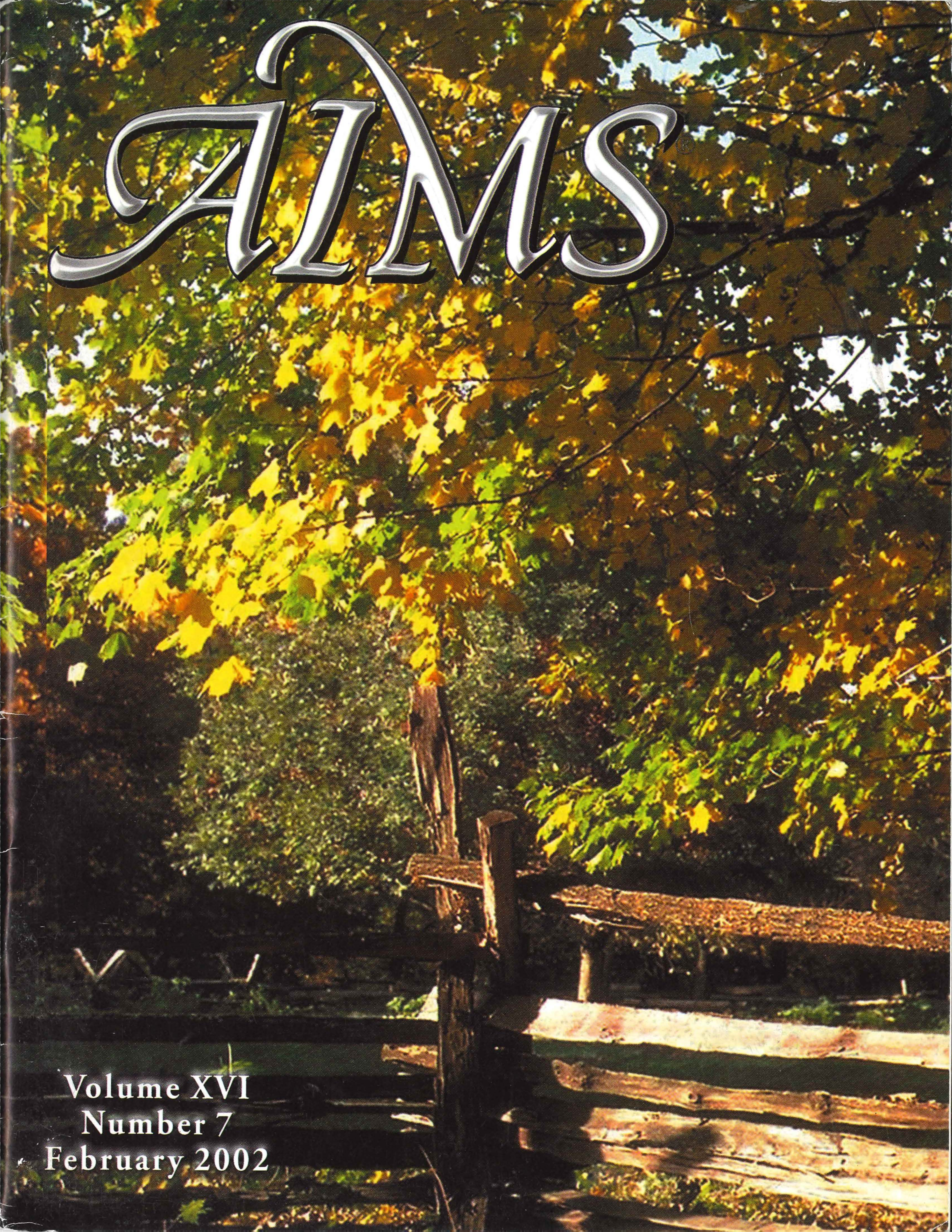
AIMS



Volume XVI
Number 7
February 2002

The **AIMS Education Foundation** is a research and development organization dedicated to the improvement of the teaching and learning of mathematics and science through a meaningful integrated approach.

AIMS®

AIMS is published 10 times a year by the **AIMS Education Foundation**, a not-for-profit corporation located at 1595 South Chestnut Avenue, Fresno, CA 93702-4706. Send address changes to P.O. Box 8120, Fresno, CA 93747-8120. Phone (888) 733-2467, Fax (559) 255-6396. Web Page: http://www.aimsedu.org
E-mail: aimsed@fresno.edu

AIMS® Magazine Subscription Information

Mail request to:
AIMS Education Foundation
P. O. Box 8120
Fresno, California 93747-8120

Please indicate your name, mailing address, grade level you teach, the volume number and quantity desired. Each volume contains 10 issues. All *AIMS* subscriptions are $30.00.

Vol. VII (1992-93)
Vol. VIII (1993-94)
Vol. IX (1994-95)
Vol. X (1995-96)
Vol. XI (1996-97)
Vol. XII (1997-98)
Vol. XIII (1998-99)
Vol. XIV (1999-00)
Vol. XV (2000-01)
Vol. XVI (2001-02)

In This Issue

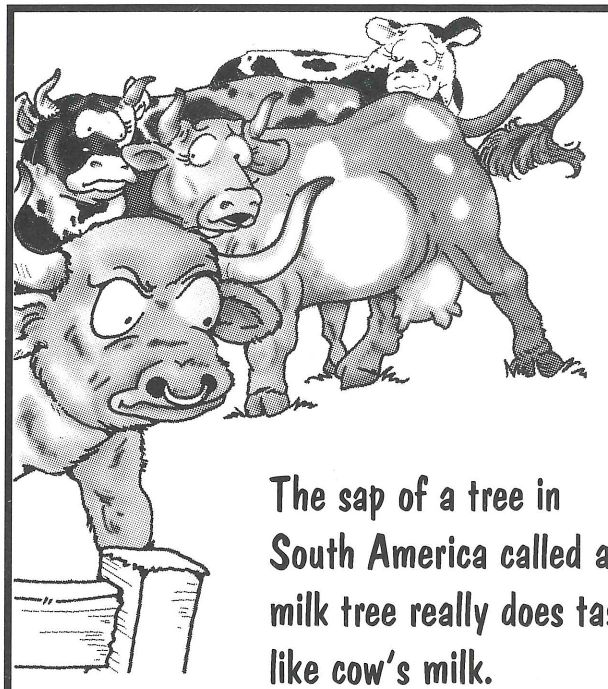
National Arbor Day is celebrated the last Friday in April. Many states, however, celebrate the day according to their best tree planting times. Check the list below and select from our magazine offerings to enhance your Earth-friendly curriculum.

States celebrating Arbor Day on the last Friday in April: Arizona, Delaware, Idaho, Illinois, Indiana, Iowa, Kansas, Massachusetts, Michigan, Minnesota, Montana, Nebraska, Nevada, New Hampshire, New Jersey, New York, Ohio, Pennsylvania, Rhode Island, South Dakota, Texas, Utah, Wisconsin

Alabama: last full week in February
Alaska: third Monday in May
Arkansas: third Monday in March
California: March 7–14
Colorado: third Friday in April
Connecticut: April 30
District of Columbia: last Friday in April
Florida: third Friday in January
Georgia: third Friday in February
Hawaii: first Friday in November
Kentucky: first Friday in April
Louisiana: third Friday in January
Maine: third full week in May
Maryland: first Wednesday in April
Mississippi: second Friday in February
Missouri: first Friday in April
New Mexico: second Friday in March
North Carolina: first Friday following March 15
North Dakota: first Friday in May
Oklahoma: last full week in March
Oregon: first full week in April
South Carolina: first Friday in December
Tennessee: first Friday in March
Vermont: first Friday in May
Virginia: second Friday in April
Washington: second Wednesday in April
West Virginia: second Friday in April
Wyoming: last Monday in April

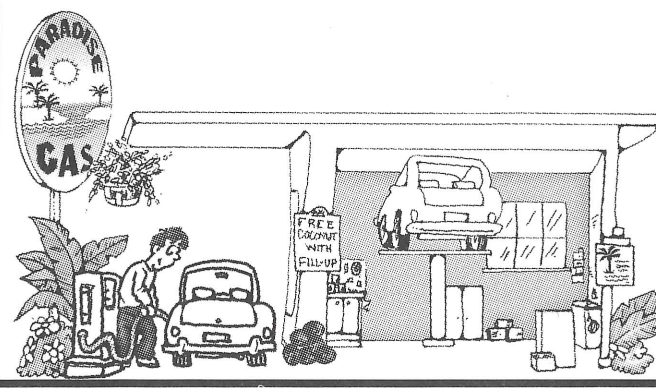
The AIMS magazine is being printed on recycled paper as a conservation effort.

Isn't It Interesting... Timber Lines

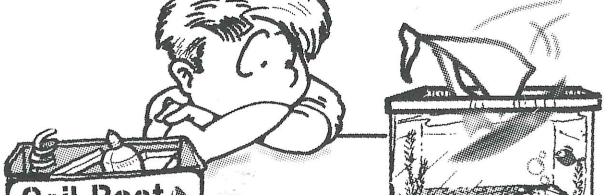


The sap of a tree in South America called a milk tree really does taste like cow's milk.

A tropical tree has sap that can be processed into a liquid very much like gasoline. Scientists are looking into the possibility of using it for fuel in the future.



The heaviest type of wood is ironwood. It is so dense that it doesn't even float.



Wood became so scarce in France during the 1300s that wooden coffins were in short supply. They were sometimes rented out and then re-used.

Compare a Tree to Me

by Sheryl Mercier and Evalyn Hoover

Topic
Looking at a tree

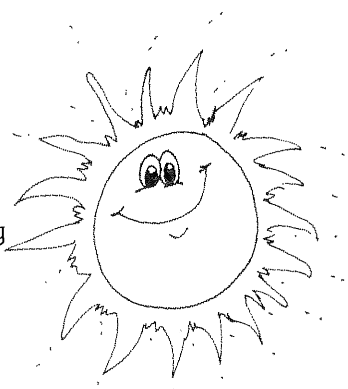
Key Question
How is a tree's structure like your own body?

Focus
Students will examine a tree to find the similarities between their body and a tree's structure.

Guiding Documents
Project 2061 Benchmark
- *Some animals and plants are alike in the way they look and in the things they do, and others are very different from one another.*

NRC Standard
- *Ask a question about objects, organisms, and events in the environment.*

Science
Life science
plants



Integrated Processes
Observing
Comparing and contrasting
Communicating

Materials
Tape measure
Paper
Crayons
Trees on the playground that the students can use to identify the structures

Background Information
There are at least 50,000 varieties of trees on Earth. Trees are living things that need food (from leaves and roots), water (from roots), light, air, and space in order to survive. Trees are important to humans for building material, beauty, shade, and enjoyment. The heights of the trees vary from 15 to 360 feet when mature. Trees are also home to many birds, insects, and animals.

Bark is the outer covering that protects the tree from insects, animals, and disease. Bark can be rough, smooth, fuzzy, deeply furrowed, or even shaggy.

Leaves are the food factories for the tree. They capture energy from the sun, combine it with chlorophyll, carbon dioxide, and water to make sugar which is their food. Leaves are broad and flat, needlelike, or made of little scales, small or large.

Trunks and branches contain tubes that move the sugars, the minerals, and water throughout the living part of the tree. The trunk provides the support for the branches, and the branches provide the support for the leaves. The outside bark protects the tree from insects and other animals, disease, and fire.

Roots absorb water from the soil and anchor the tree in the ground. The water contains dissolved minerals (nutrients) needed by the tree. Most trees have a taproot that grows straight down, lateral roots that spread out over a wide area, smaller rootlets and finally root hairs that absorb 95% of the water.

Through this lesson, the students will become more aware of trees and the structure of trees by comparing the structure of a tree to their own body structure.

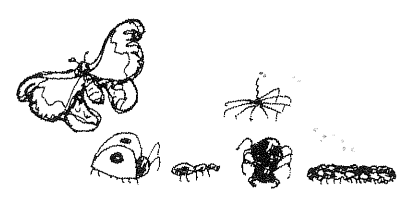
Management
1. Beforehand identify the trees on the playground that can be used for this activity. It is suggested that you pick deciduous, broad–leaved trees if possible. Locate several different kinds of tree for the students to observe. If there are no trees in your area, then bring a live potted tree to class or collect pictures of trees for the students to use in their observations.
2. The accompanying student pages can be utilized by the students if appropriate, or they can serve as ideas for comparison charts for contributions from the whole class.

Procedure
1. Brainstorm with the students where they have seen trees. Give them a piece of drawing paper and have them draw a tree from memory. Save their drawings for later. Have them describe the tree they drew.

2. Tell students that now they are going outside to take a closer look at trees. Take the students out to observe the trees that are growing on the playground. If you can, identify the trees by name for the students.
3. Choose one tree to observe as a class so that you can model how to observe a tree.
4. Pick up some leaves from the tree and give them to the students to examine carefully. Ask them questions that lead them in their observations.
   - How big is your leaf?
   - How does it compare with the size of your hand?
   - What is the color of the leaf?
   - Describe the shape of the leaf.
   - Does the leaf look the same on both sides?
5. Suggest that the students hold out the leaf and their hand at the same time. Tell them to compare the leaf to their hand. Ask them if they can see that some leaves have edges that look like their fingers?
6. Tell them to look at the veins in the leaf and the veins in their hand (or an adult's hand). Ask them if they know the purpose of the veins in their hands. Discuss with the student that the veins in their hands carry blood and nutrients to their hands and fingers. Ask them what they think is carried by the veins of a leaf. Explain that the veins in a leaf carry water and food to the leaves.
7. Now invite the students to examine the bark of the tree. Encourage them to rub their hands over the bark of their tree. Tell them the bark protects the inside of a tree and keeps all the tree's food and water inside. Then tell the students to rub their hand up and down their own arm and feel their skin. Their skin protects their veins, muscles, and bones inside them too.
   - Describe the bark of the tree, its color, texture, and smell.
   - Do a rubbing of the bark of the tree.
   - How many colors can you see on the bark of the tree?
   - Compare the colors of the bark to the colors in your skin.
   - Is the bark of the tree thick or thin? Is it thicker or thinner than your skin?
8. Instruct the students to stand under the tree, lift their arms high above their heads, and look up. Tell them to notice that the branches are like their arms. One difference between their arms and the tree's branches is that the tree most often has several branches at different heights. Ask the students if there are any branches that are the same size as their arms. Tell them to guess how big around the branches are.
9. Challenge the students to stand up straight and tall with their arms to their sides and their feet together. Ask them what is holding them up, (their legs). Tell them that the trunk of the tree is similar to their body and holds up the branches and leaves.
10. Finally, tell them to look at their feet. Their feet support their legs and the rest of their body and keep them from falling down. Tell the students that way down underneath the ground the tree grows roots. The roots dig into the Earth and give a tree the strength it needs to stand.
11. Ask the students if they are as big as the tree. Tell them to take a tape measure and measure a tree's trunk to see how big around it is. Then encourage them to hold hands with some of their friends and see how many it takes to go around a tree.
12. Encourage the students to look around and find a favorite tree to observe in the same manner that you did as a class (suggest that they choose a deciduous, broad-leaved tree). They should move close and feel the bark of the tree, the leaves, and the branches. Instruct them to make the same observations that they did as a whole group.
13. Have children return inside and now draw their tree from memory. Put the pairs of drawings side by side around the room. Take an art walk and ask students to compare the drawings. Are they more accurate? Do they contain more details? Are they more colorful?

Discussion

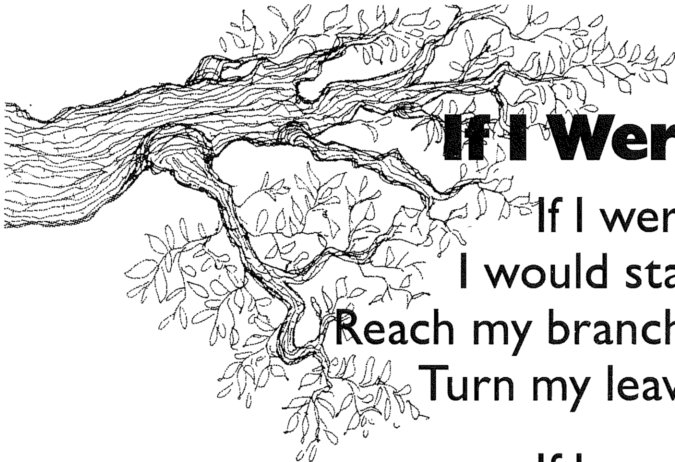
1. Describe your tree so well that someone who is not acquainted with your schoolyard can identify which tree is yours.
2. How are you like a tree? How are you different from a tree?
3. What is the most important thing you learned about your tree?
4. What color are the leaves of the tree? ...the bark of the tree?
5. What does the bark smell like?
6. What holds the tree up?
7. Compare the bark of various trees as to texture, color, and thickness?



Extensions

1. Suggest to the students that they write a story about their tree. Tell them to describe what it looks like, what it needed to make it grow as big as it is, and describe the leaves and bark of the tree.
2. While you are outside under a tree, have children use their bodies to bend and twist like branches, flutter around like leaves, stand straight and tall like a tree trunk, make noise like wind in the leaves, stretch and make shade, catch sunlight with their leaves, reach deep in the soil with their roots.
3. Develop vocabulary and encourage discussions by making a comparison chart of their observations.

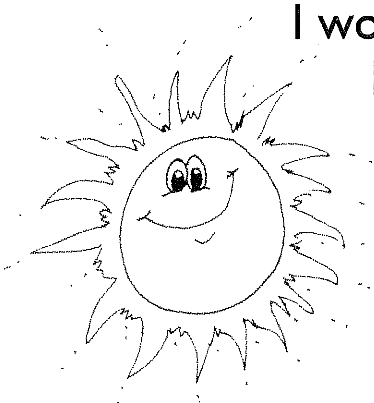
Tree	Me
• Has leaves that make food from sun, air and water	• Eat foods my family prepares
• Stays in one place because roots hold it up	• Can move and walk around on my feet
• Have branches that hold out the leaves	• Have arms that move my hands
• Has a trunk that supports the tree	• Have a body with a skeleton to support me
• Has bark that protects the living part from animals, insects, and disease	• Have skin that protects me inside from germs and disease
• Needs food, light, water, air, and space to survive	• Need food, water, clothing, air, shelter, and love to survive
• Has tubes that carry the food, water, and nutrients to all parts of the living tree	• Have a blood system of arteries and veins that carries food, water, and wastes
• Has a crown on top	• Have a head on top of my body
• Leaves and branches make noise in the wind	• I can talk, smell, see, feel, taste, and hear
• Each tree is unique	• I am unique



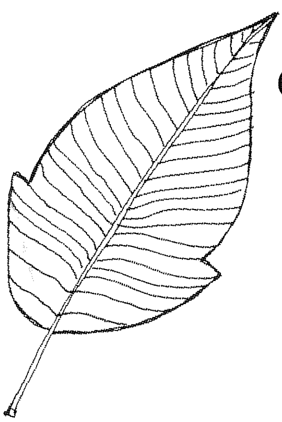
If I Were a Tree

If I were a tree,
I would stand very tall,
Reach my branches to the sky, and
Turn my leaves to the sun.

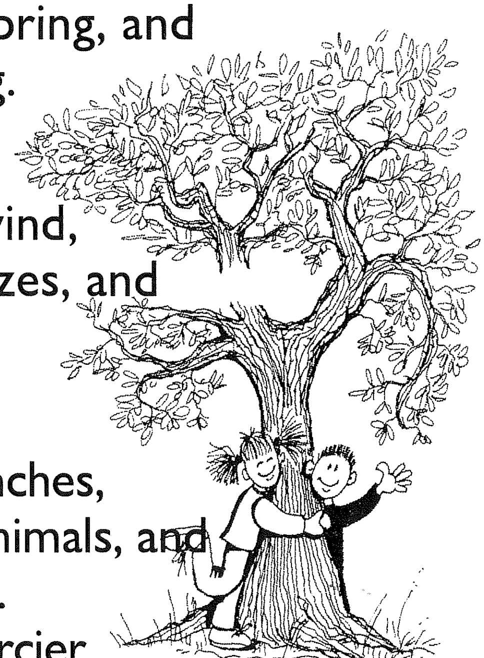
If I were a tree,
I would grow my roots into the Earth,
Drink water from the soil, and
Stand very strong.



If I were a tree,
I would capture the sunlight,
Use it with water and air, and
Make my own food.



If I were a tree,
I would drop leaves in fall,
Grow branches and leaves in spring, and
Grow all summer long.



If I were a tree,
I would dance with the wind,
Sing leaf songs with the breezes, and
Bend with the gusts.

If I were a tree,
I would spread out my branches,
Provide homes for insects and animals, and
Give beauty to people.

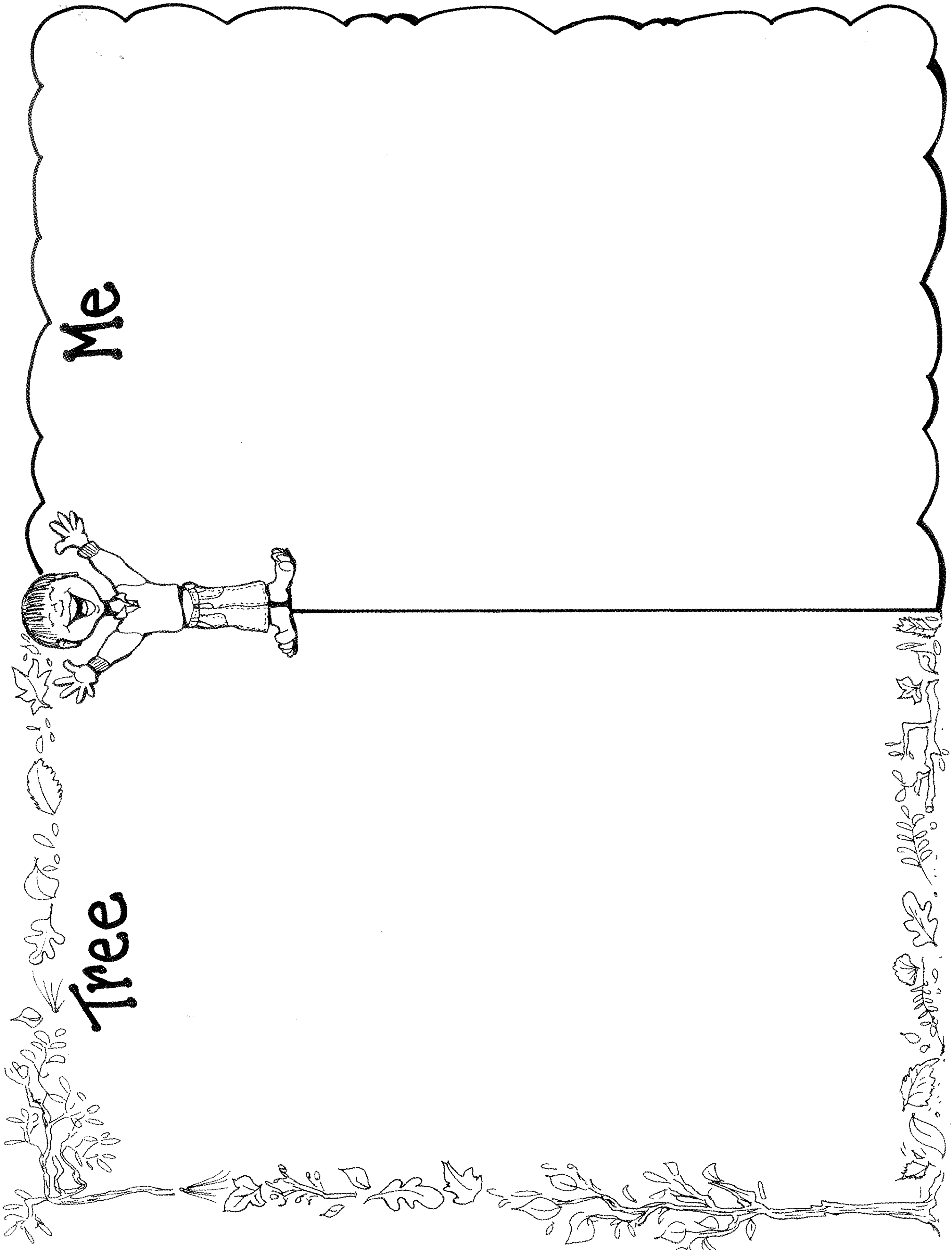
—Sheryl Mercier

Tree Leaf			My Hand		
smells	feels	looks	smells	feels	looks

Tree Bark

My Skin

smells	feels	looks	smells	feels	looks


Me
Tree

DETERMINING DIAMETERS

by Suzy Gazlay

Topic
Measuring the diameter of a tree

Key Question
How can the diameter of a tree be measured without cutting down the tree?

Learning Goals
Students will:
1. experience using a logger's diameter tape to find the diameter of a tree, and
2. apply the relationship between the circumference and the diameter of a circle to the design and effectiveness of a diameter tape.

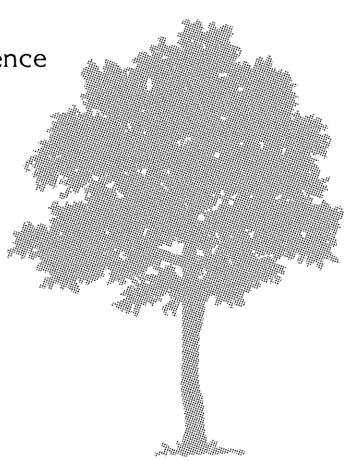
Guiding Documents
NRC Standards
- *Resources are things that we get from the living and nonliving environment to meet the needs and wants of a population.*
- *Some resources are basic materials, such as air, water, and soil; some are produced from basic resources, such as food, fuel, and building materials; and some resources are nonmaterial, such as quiet places, beauty, security, and safety.*

*NCTM Standards 2000**
- *Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life*
- *Understand that measurements are approximations and how differences in units affect precision*
- *Select and use benchmarks to estimate measurements*

Math
Geometry
   diameter and circumference
Measurement
Estimation

Science
Life science
   trees
   natural resources

Integrated Processes
Observing
Classifying
Predicting
Collecting and recording data
Comparing and contrasting
Interpreting data



Materials
For the class:
   piece of wood or cardboard measuring one board foot (see *Background Information)*
   large cylindrical garbage can or other similar object
   yardstick

For each student:
   logger diameter tape templates
   clear tape or white glue
   tape measure or string cut to 4.5 feet
   several pieces of masking tape

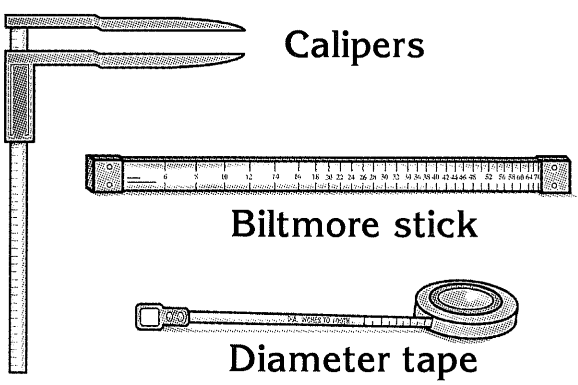
Background Information
Trees are valued for many different and excellent reasons. In addition to their beauty and all that they do for the environment, people throughout history have used wood for building materials, fuel, and many other purposes. Such familiar items as paper and cardboard are made from wood, as are many other products. Some countries do not have trees, so they must import wood or wood products, or use alternatives. Wood and wood products are widely used in the United States. It's difficult for us to imagine what our lives would be like without wood.

When most people describe the dimensions of a tree, they are likely to consider first its height, and then perhaps its circumference. A logger, who looks at a tree for its potential to produce lumber, sizes it up in terms of estimated *board feet.* A board foot is a piece of wood with dimensions that total 144 cubic inches. (One possibility would be a piece of wood measuring 12 inches high, 12 inches wide, and one inch thick.) The tree would be described according to its board foot volume: how many board feet of lumber can be cut from it. Of course there are variables such as how the tree is cut, the dimensions of the lumber, how much is wasted, and the skill and efficiency of the logger and the workers in the sawmill. So, although the board foot measurement is in widespread use in the United States, it is understood to be an estimation at best.

The diameter of a tree is one of the most important measurement needed to determine board feet. The base of a tree may be considerably wider than

the main part of the trunk, so the measurement is taken farther up the trunk. The common standard is the *diameter at breast height*, or DBH. Such a measurement would vary greatly if it depended upon the actual height of the logger, so the DBH has been standardized to be 4.5 feet (1.37 meters) above the forest floor. It includes the outside bark. DBH is measured to the nearest 0.1 inch but is rounded to the nearest inch for recording and calculating. If the tree is growing straight on a hill, the DBH is measured from the uphill side. If the tree forks below the DBH, each trunk is measured as a separate tree. If *duff* (a layer of dead leaves and/or needles and perhaps other decomposing plant material) is present, it is considered to be part of the forest floor and is the base of the measurement. Branches, pieces of wood, other woody debris, and above ground roots are not part of the distance measured.

It would be a challenge to directly measure the actual diameter of a tree. Instead, loggers have a choice of mathematically calibrated tools that are designed to indicate the length of the diameter, even though the diameter itself is not measured directly.



Tree calipers are made of wood or aluminum in a couple of different styles. The caliper's prongs are pressed against opposite sides of the tree at the DBH. The distance between them is measured according to a calibrated scale. Calipers are best for measuring smaller trees. A *Biltmore stick* is a simple hand-held tool that uses the mathematics of distance, angle, and proportion to estimate—rather than to measure—the height. Held sideways, it can also be used to approximate the diameter. Flexible *diameter tapes*, usually made of cloth, fiberglass, or steel, are marked off in 3.14-inch intervals. The diameter of the tree is determined by encircling the tree with the tape and noting the number on the tape where the circle is complete. Because of the calibration on the tape, this number indicates the diameter rather than the circumference. The intervals on the tape are 3.14 inches, the value of pi (π). This calibration directly applies the formula for determining the diameter when the length of the circumference is known: $d = c/\pi$.

Other important tree measurements include the height (ground to tip, dead or alive), and the *merchantable height*, which is the height of the part of the tree—usually the trunk—from which the desired product can be made. Merchantable height does not include the stump (usually about two feet) or any part of the trunk that is hollow, rotten, or excessively crooked. The usable part is typically described in terms of 16-foot *logs* and 8-foot *half logs*.

If you wish to extend this lesson to include determining the board feet in the trees used, you will also need to measure, or at least estimate, the merchantable height in terms of "logs." Two methods or "rules" are commonly used, the Doyle rule and the International $\frac{1}{4}$ inch rule. Tables showing board feet according to DBH and number of logs can be found in forestry reference materials and on the Internet (see *Resources*).

Management

1. Diameter tapes are most accurate on trees with approximately round trunks.
2. Refer to the *Background Information* to show the students how to determine the level at which to measure the tree. Two other guidelines to consider: if the tree is leaning, measure the diameter on a plane perpendicular to the "lean" of the tree rather than parallel to the ground; if there are branches or other projections at the DBH location, place the tape above them but as close to the DBH as possible.
3. Advise the students to measure the tree with the numbers on the tape right side up to avoid errors stemming from trying to read the scale upside down.
4. To assemble the diameter tape, cut the sections of tape apart from both pages. Place the tab at the end of one section under the space of the next sequential number and line up the gradation. Secure either with clear tape or white glue rather than a glue stick or paste. Tapes may be laminated so that they last longer.
5. The tape provided in this activity will measure circumferences of slightly over 110 inches, which is a fairly large tree. If you are measuring massive trees with broad trunks and the tape is too short, it can easily be extended by joining other tapes together at the last gradation of one and the first gradation of the second and renumbering the intervals on the second tape.
6. To save time, you may wish to cut out the strips using a paper cutter and just have the students tape or glue them together.

7. If it is difficult to get a piece of wood to demonstrate a board foot, cut several sheets of heavy cardboard to measure 12" by 12" and glue them together to a thickness of one inch.
8. Students should already have a basic understanding of the relationship between the diameter and the circumference of a circle.
9. Although the actual measuring of a tree is easiest when the students work in pairs, it is recommended that each student make his or her own diameter tape to keep and use at home.

Procedure

1. Ask each group to make a list of the different ways they and their family use wood. Compile the information in a class list, noting which uses are mentioned most often and which are mentioned but a few times. Go through the list and discuss what alternatives, if any, could be used if wood were not available.
2. Find out what the students already know about how wood is produced. Help them trace the steps backwards from the finished product to the living tree. Ask the students how they think a logger or lot owner might be able to figure out how much usable wood is in a tree—before it is cut down.
3. Explain what constitutes a board foot, using an example (see *Management 5*). Work together to figure out other dimensions which would also be considered a board foot. [e.g., 9 x 8 x 2; 6 x 8 x 3; 12 x 4 x 3, etc.] Look around the room and estimate how many board feet would be in each of several different objects. Objects with straight sides such as boxes are easiest for students to visualize.
4. Explain to the students that a logger needs two measurements in order to estimate how much usable wood is in a tree: the merchantable height and the diameter at breast height or DBH, explaining what DBH means (see *Background Information*). Ask them how someone could accurately measure the diameter when they can't get to it directly. Discuss their responses in terms of how efficient each method would be if a logger needed to measure a large number of trees in a short time.
5. Describe some of the tools a logger uses (see *Background Information*). Tell the students that they are going to make a diameter tape to find the diameter of trees on the playground or in the neighborhood.
6. Distribute templates to each student and demonstrate how to assemble the tape.
7. Ask the students what they notice about the gradation on the tape [spaces are too wide to be inches]. Place a garbage can or other large cylinder on a table so that a point somewhere in the middle section is around DBH. Make a point of locating the DBH and marking it with a small piece of masking tape. Select a pair of students to demonstrate how the tape is used by wrapping it around the garbage can (or other large cylinder) until the front end of the tape can touch the rest of the tape. Read the number on the tape where the end touches it. Have several other teams see if they come up with the same number. Ask how the accuracy could be checked, guiding the discussion to measuring the actual diameter itself, since it is accessible on the garbage can. Use a standard measure such as a yardstick, NOT the diameter tape. The length of the diameter should be very close to what was determined using the diameter tape.
8. Challenge the class to figure out how a diameter tape can use increments different than an inch to show the length of the diameter in inches. If necessary, bring the discussion around to thinking about what geometric part of the can or tree was measured [circumference]; what they already know about how to find the diameter and circumference of a circle; and what the relationship is between the two. [$c = \pi d$; $d = c/\pi$] Direct them to use a standard scale ruler to measure the intervals. Guide them to understand why it is important for the intervals to be consistently 3.14 inches.
9. Give each pair of students a recording sheet and go to an area with trees. Encourage the students to measure trees of different sizes if possible, recording the DBH and the general location of each.
10. Back in class, compare the data and discuss the results.
11. Optional: measure or estimate the merchantable height and use the data table found at http://ohioline.osu.edu/for-fact/0035.html to determine the number of board feet for one or several trees (see *Background Information)*.

Discussion

1. What else could your diameter tape be used to measure besides trees?
2. Would the diameter tape be an accurate measure on all trees? Explain your answer. [trunk DBH needs to be relatively circular]
3. Why do you think 4.5 feet was selected to be the standard DBH?
4. How accurate do you think diameter tape measurements are, and why? What could make them even more accurate?
5. Can you think of any way to measure a tree's diameter that might be more efficient than a diameter tape? ...more accurate? ...both?

6. What would you estimate to be the diameter of your leg just above the knee? ...your arm just below the elbow? ...your waist? Use your tape to find out. Ask a friend to help you measure the diameter of your head. Do you think the results will be the same measuring from ear to ear as from the bridge of your nose to the back of your head? Why or why not? What did you find?

Extensions

1. Learn how to make and use a Biltmore stick. Compare its use and accuracy to your diameter tape.
2. Research to learn more about the logging process and what happens to a log from the time a tree is cut until the lumber from it arrives at a lumberyard, or the pulp is made into paper.
3. Logging can be a controversial issue when the trees in question are integral parts of the habitat of a threatened or endangered species, or when there are other reasons to consider protecting the forest, especially when "first growth" trees are involved. Research to find as many different points of view as possible, including loggers and mill workers, lumber companies, advocates of tree farms, environmentalists, foresters, and so forth. Analyze the information you find. Try to identify the strengths and weaknesses of each perspective. Older students may wish to role-play a discussion representing each point of view. Consider inviting proponents of several different positions to speak to the class.
4. Design an investigation to determine if tree diameter is related to the height of a tree.
5. Use field guides, tree books, Internet sites, or other resources to identify the trees. Arrange data according to tree types. Look for patterns to compare the different species.

Evidence of Learning

1. Check to see that students accurately use the logger's diameter tape.
2. Listen as students explain why using a logger's diameter tape illustrates the geometric relationship between the diameter and circumference of a circle.

Curriculum Correlation

Literature

For young readers:

Tonsgard, William R. *Willie and Sam*. Gulliver Books. Juneau, Alaska. 1999.
Life in an Alaskan logging camp from a child's point of view.

For older readers:

George, Jean Craighead. *There's an Owl in the Shower*. HarperCollins Juvenile Books. New York. 1995.
The son of an out-of-work logger inadvertently brings home an owlet of the very species that has halted logging and cost his father his job.

Gintzler, A.S. *Rough and Ready Loggers* (Rough and Ready Series). Fitzgerald Books. Bethany, MO. 1998.
Stories of Western loggers at the height of the timber industry.

Nelson, Sharlene and Ted W. Nelson. *Bull Whackers to Whistle Punks: Logging in the Old West (First Books—Western U.S. History Series)*. Franklin Watts, Incorporated. New York. 1996.
Logging in the West during the late 1800s and early 1900s: daily routine, jobs, tools and machinery.

Smith, Roland, and Michael J. Schmidt. *In the Forest with the Elephants*. Harcourt Brace. San Diego. 1998.
A day in the life of carefully trained elephants and their riders who log the teak forests of Myanmar in Southeast Asia.

For all ages:

Appelbaum, Diana. *Giants in the Land*. Houghton Mifflin Co. Boston. 1993.
The 17th century process of logging the giant white pines of the forests of New England for masts for British warships.

Resources:

Project Learning Tree. American Forest Foundation, 1111 19th Street NW, Washington, DC, 20036. 1993. (see *Internet* for website)

Internet

Pictures of the process of using a diameter tape
http://www.cnr.vt.edu/dendro/forsite/dtape.htm

Determining board feet
http://ohioline.osu.edu/for-fact/0035.html

Biltmore stick
http://www.cnr.berkeley.edu/departments/espm/extension/TREESTK.HTM

Project Learning Tree
http://www.plt.org/html/about_plt/about_index.html

Home Link

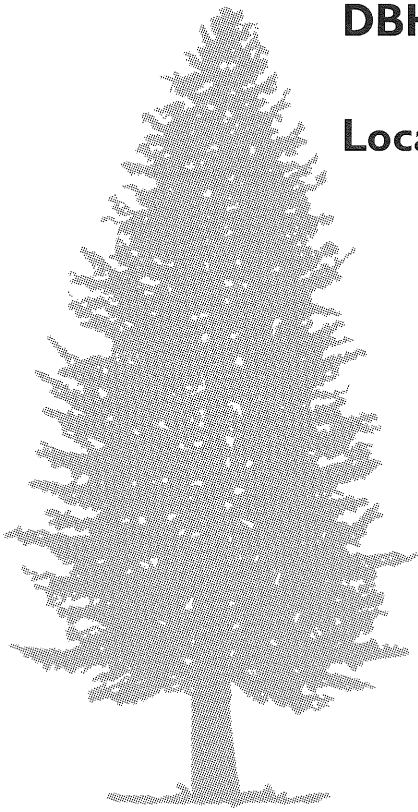
Have the students take their diameter tapes home, demonstrate them to their family, and measure several trees in their yards or neighborhood.

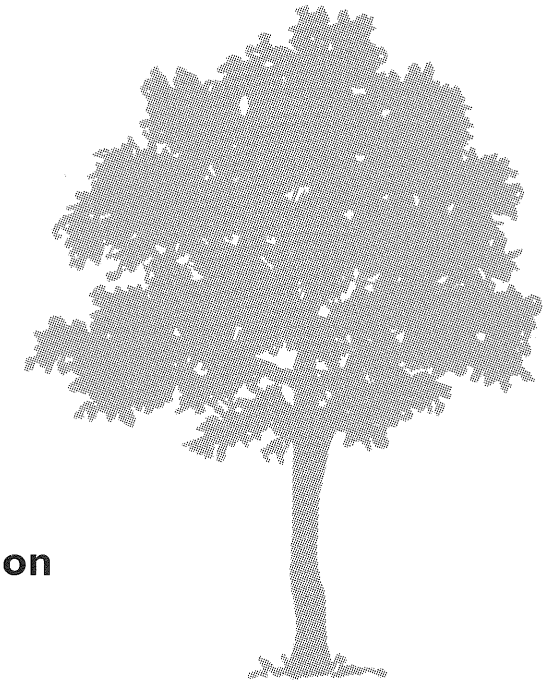
* Reprinted with permission from *Principles and Standards for School Mathematics*, 2000 by the National Council of Teachers of Mathematics. All rights reserved.

DETERMINING DIAMETERS

DBH

Location



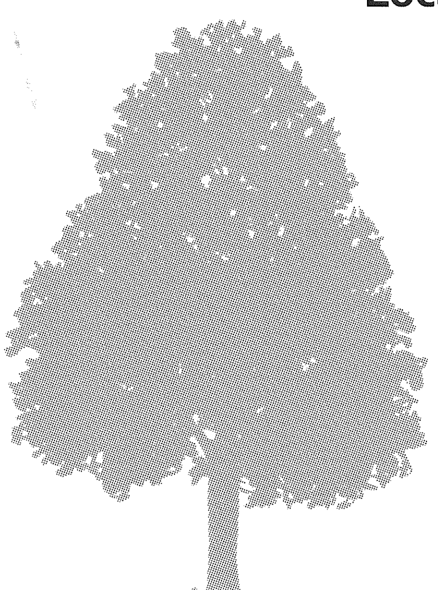


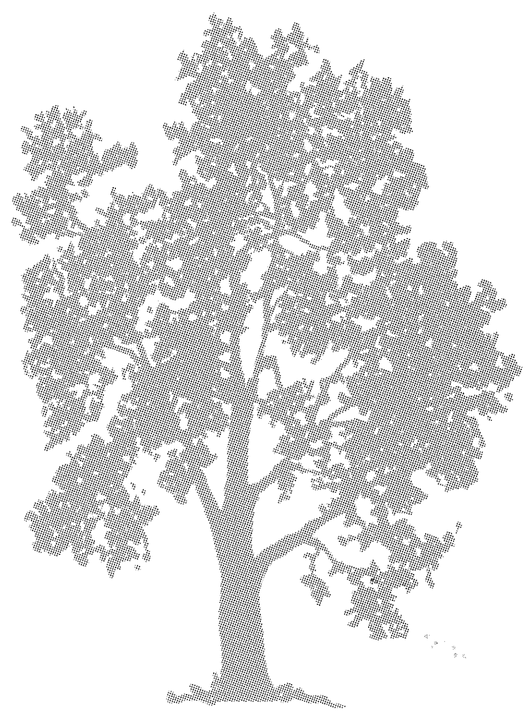
DBH

Location

DBH

Location





DBH

Location

TAB	16	17	18
TAB	13	14	15
TAB	10	11	12
TAB	7	8	9
TAB	4	5	6
	1	2	3

TAB	34	35	36
TAB	31	32	33
TAB	28	29	30
TAB	25	26	27
TAB	22	23	24
TAB	19	20	21

PATTERNS of GROWTH

by Sheldon Erickson

Topic
Tree Growth/Algebraic Thinking

Learning Goals
Students will:
- construct graphs comparing the growth rates of trees,
- be introduced to constant and changing rates of growth, and
- develop equations to predict the age and the height of a tree from its diameter.

Key Questions
1. How could you predict a tree's height from its age?
2. How could you predict the thickness (diameter) of a tree from its age?
3. How can you predict the height of a tree from its thickness (diameter)?

Guiding Documents
Project 2061 Benchmarks
- *In all environments—freshwater, marine, forest, desert, grassland, mountain, and others—organisms with similar needs may compete with one another for resources, including food, space, water, air, and shelter. In any particular environment, the growth and survival of organisms depend on the physical conditions.*
- *Mathematical statements can be used to describe how one quantity changes when another changes. Rates of change can be computed from magnitudes and vice versa.*
- *Graphs can show a variety of possible relationships between two variables. As one variable increases uniformly, the other may do one of the following: always keep the same proportion to the first, increase or decrease steadily, increase or decrease faster and faster, get closer and closer to some limiting value, reach some intermediate maximum or minimum, alternately increase and decrease indefinitely, increase and decrease in steps, or do something different from any of these.*

NRC Standards
- *Use appropriate tools and techniques to gather, analyze, and interpret data.*
- *Develop descriptions, explanations, predictions, and models using evidence.*

*NCTM Standards 2000**
- *Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations*
- *Model and solve contextualized problems using various representations, such as graphs, tables, and equations*
- *Use graphs to analyze the nature of changes in quantities in linear relationships*
- *Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit*

Math
Algebraic thinking
graphing
developing linear equations

Science
Life science
botany
tree growth

Integrated Processes
Observing
Collecting and organizing data
Predicting
Inferring
Interpreting data

Materials
Student pages

Background Information

Each species of tree has a growing pattern. Genetic coding determines the physical nature of a tree which affects the general rate of growth, terminal height, and physical proportions. Although all trees of a given species tend to grow in a similar pattern and range, individual differences occur for a number of environmental factors including climatic conditions (rainfall, temperature, daylight), competition, disease, and infection.

The variation in tree measurements within a species makes it more difficult for scientists to study growth. The measurements of a typical tree must be developed by averaging the measurements of a stand of trees. In a few commercial forests, the growth of

trees can be studied over a long period of time. Most forests are so slow growing that a longitudinal study of this type is impractical. Most data are gathered by measuring a large number of trees in one stand and then finding the average measurements of the trees of the same age. By gathering data from stands of trees in different environments and regions, comparisons can be made.

To standardize the measures of the diameter of a tree, it is measured at chest (breast) height of an adult (4.5 ft or 1.37 m). This standard of measure is often listed as DBH (Diameter at Breast Height) in forestry literature.

Since the objective of this investigation is to determine the general growth patterns of a species and compare it to other species, the most broadly based average was used for each species.

By graphing the data, one can quickly get a picture of the growth patterns of different species. If the data points lie along a straight line (are linear), the rate of change in the two variables is consistent. If a crooked line results from the data points, the rate of change in the two variables is inconsistent. If the data points form a smooth curve, the rate of change is decreasing or increasing at a consistent rate.

As students make the graphs for all four trees for height to age, they will recognize that they all form relatively smooth curves. (Ash is less regular.) This pattern should draw students to the conclusion that all trees grow slower as they grow older. The steepness of the curves also allows students to see that rate of growth. Students should interpret that a poplar tree grows much more rapidly than a beech.

When comparing the diameter of a tree to its age, two trees (ash, lodgepole pine) have inconsistent data. The data of the beech tree form a nearly straight line. Each year the tree's diameter grows nearly the same amount. In 230 years (250 – 20), a beech tree grows an average of 19.2 inches in diameter (19.9 – 0.7). Divided evenly, that means each year the diameter grows by approximately 0.083 inches. This rate of change, 0.083 inches per year, is the slope of the line on the graph. This consistent rate allows us to easily develop an equation predicting the diameter of the tree. In the first 20 years, a beech tree grows to a diameter of 0.7 inches. After that, it grows at a rate of 0.083 inches per year. If the age in years is represented by A, then the equation might be written:

$$\text{Diameter} = 0.083(A-20) + 0.7$$

Using the equation and working backwards, a student can predict the age of a beech tree with a known diameter of say 18 inches. Seven-tenths inches of the diameter happened in the first 20 years, so 16.3 inches of growth happened in the remaining time. Since the diameter grows by 0.083 inches each year, dividing the 16.3 inches by 0.083 tells us the year for that amount of growth, ($16.3 \div 0.083 \approx 196$) 196 years. Adding back the original 20 years gives a total age of approximately 216 years.

The poplar's data produce a smooth curve communicating that as a poplar grows older, it adds less and less to its diameter each year. The smoothness of the curve suggests that the rate is very consistently decreasing. The initial steepness shows it starts growing at a much quicker rate than a beech tree. In the end, the slope of the poplar's curve is less steep than the beech tree's line, showing that the poplar finally grows more slowly than a beech tree. The equation of the poplar's curve is much more difficult to develop. It is quadratic in nature and is more appropriately considered in advanced algebra.

A graph comparing the height to diameter of trees provides inconsistent graphs again of the lodgepole pine and the ash. In this graph the poplar data produces a straight line. For 81 feet of height (142-61=81), there is an increase of 13.9 inches of diameter (23.7-9.8=13.9). That means for each increase of an inch of diameter after 9.8 inches there is an increase of height 13.9 feet. There is a ratio of 13.9 ft/in. An equation for the height of a tree can be written:

Let H = height of tree in feet
Let D = diameter of tree in inches
$H = 13.9(D-9.8) + 61$

The height of a typical poplar tree can be predicted by substituting the diameter of the tree into the equation and solving for the height.

The graph of the beech tree forms a smooth curve. The curve communicates that as the tree gets higher, it needs to increase the diameter in greater amounts to support the mass of the tree.

Differences in the graphs comparing height to diameter for the poplar and the beech can be understood the general appearance of the tree and its growth habits are considered. The poplar is a very straight tall tree, clear of lateral branches for a considerable height. Its slender growth pattern does not require a large increase in diameter for stability. The beech tree has a relatively short thick trunck with large low spreading limbs. The beech increases in mass by lateral growth as well as vertical. The mass of the tree grows exponentially and must be supported by a much thicker trunk.

Management

1. Before beginning this investigation determine what graphs are most appropriate for students. The American beech and yellow poplar data provide the most consistent graphs. The lodgepole pine and white ash were included to provide a greater variety of species.
2. Determine whether the class should use metric or customary units of measure.

Procedure

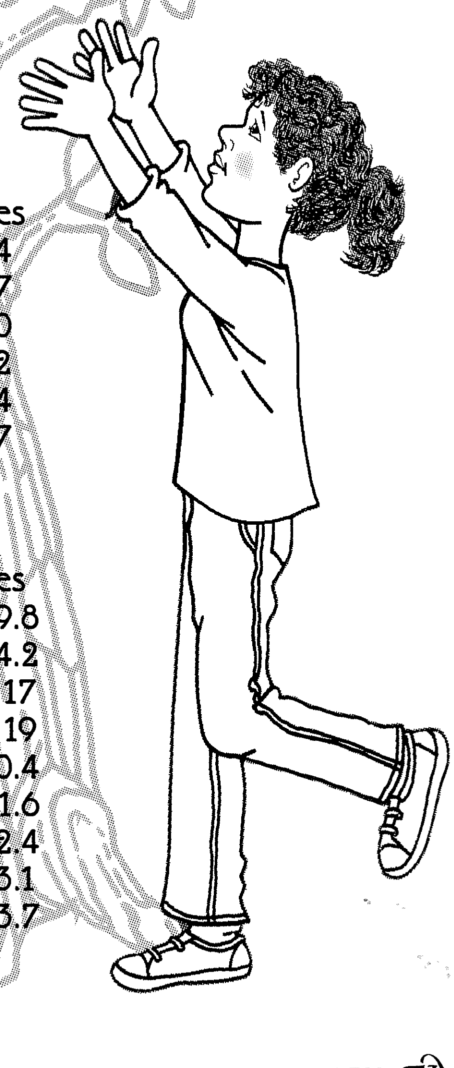
1. Ask the first *Key Question* and have students discuss what they think the relationship of height to age of a tree is and how it might differ between species.
2. Distribute the data sheet and graph and have the students graph the data for each species.
3. Have the students compare and contrast the graphs for each species of tree and interpret what the graphs tell them about the trees.
4. Discuss the second *Key Question* with students. Have them consider how they think the graphs will differ.
5. Distribute the graph and instruct the students to graph the data of diameter to age.
6. Have the students compare and contrast the graphs for each species of tree and interpret what the graphs tell them about the trees.
7. If appropriate, have students develop an equation relating the diameter to age of a beech tree.
8. Discuss the third *Key Question* with students. Have them discuss how they anticipate the graphs will appear.
9. Distribute the graph and instruct the students to graph the data of height to diameter.
10. Have the students compare and contrast the graphs for each species of tree and interpret what the graphs tell them about the trees.
11. If appropriate, have students develop an equation relating the height to the diameter of a yellow poplar tree.
12. Describe the growth habits of the poplar and beech trees (see *Background Information)* and ask students to consider how the growth habits can be seen in the shape of the graphs.

Discussion

1. How are the shapes of the graphs for heights of the four trees similar and different? [all curves, different heights, slopes, lengths]
2. Interpret what these similarities and differences tell you about tree growth. [Although trees differ in their final height, growth rates, and longevity, all trees grow more quickly in their early years and grow less as they grow older.]
3. Describe the shapes of the graphs for diameters of the four species of trees. [ash and logdepole, irregular; poplar, curved; beech, linear]
4. Interpret what the shape of the poplar's and beech's graphs tell you about their growth patterns. [the diameter of the poplar increases less and less each year, the diameter of the beech tree grows the same amount each year]
5. How can you develop an equation predicting the diameter of a beech tree from its age? (See *Background Information.)*
6. Explain how you would use your equation to predict the age of a beech tree that is 18 inches (45.7 cm) in diameter. (See *Background Information.)*
7. Describe the shapes of the graphs for heights to diameters of the four species of trees. [ash and logdepole, irregular; poplar, linear; beech, curved]
8. Interpret what the shape of the poplar's and beech's graphs tell you about their growth patterns. [for the poplar, the height and diameter increase together at a constant rate; as the beech tree gets taller, the diameter increases at a greater rate.}
9. How can you develop an equation predicting the height of a poplar tree from its diameter? (See *Background Information.)*
10. How do the growing habits of the poplar and beech trees show up in their height to diameter graphs? (See *Background Information.)*

Extensions

1. Get growth data for other species of trees and see how they compare to the four on the data sheet. Information can be found on the following web site: http://www.na.fs.fed.us/spfo/pubs/silvics_manual/table_of_contents.htm
2. If one of the four trees grows near the school site, gets its height and diameter measures and predict the trees age.



AMERICAN BEECH

	HEIGHT		DBH*	
Age	Meters	Feet	Centimeters	Inches
20	4	13	2	0.7
40	8.5	28	6	2.3
60	11.9	39	10	3.8
80	14.6	48	14	5.4
100	17.4	57	18	7.1
150	22.9	75	29	11.5
200	25.6	84	40	15.7
250	26.8	88	51	19.9

LODGEPOLE PINE

	HEIGHT		DBH*	
Age	Meters	Feet	Centimeters	Inches
20	5.5	18	8.6	3.4
50	12.5	41	16.5	6.5
80	18	59	20.6	8.1
110	22.3	73	23.6	9.3
140	25.3	83	26.7	10.5

WHITE ASH

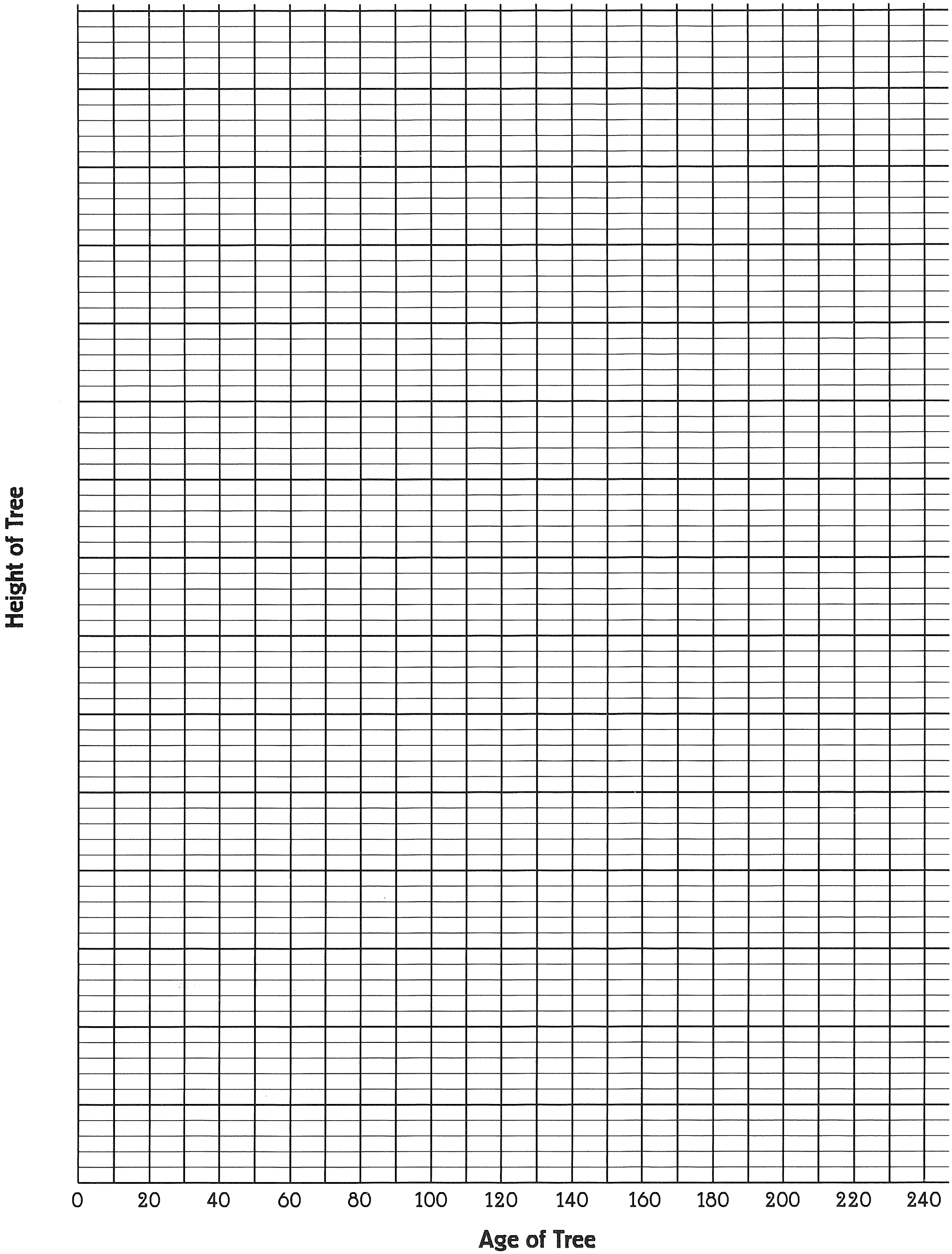
	HEIGHT		DBH*	
Age	Meters	Feet	Centimeters	Inches
20	12	39	10	4
30	17	56	18	7
40	21	69	25	10
50	23	75	30	12
60	25	82	36	14
70	27	89	43	17

YELLOW POPLAR

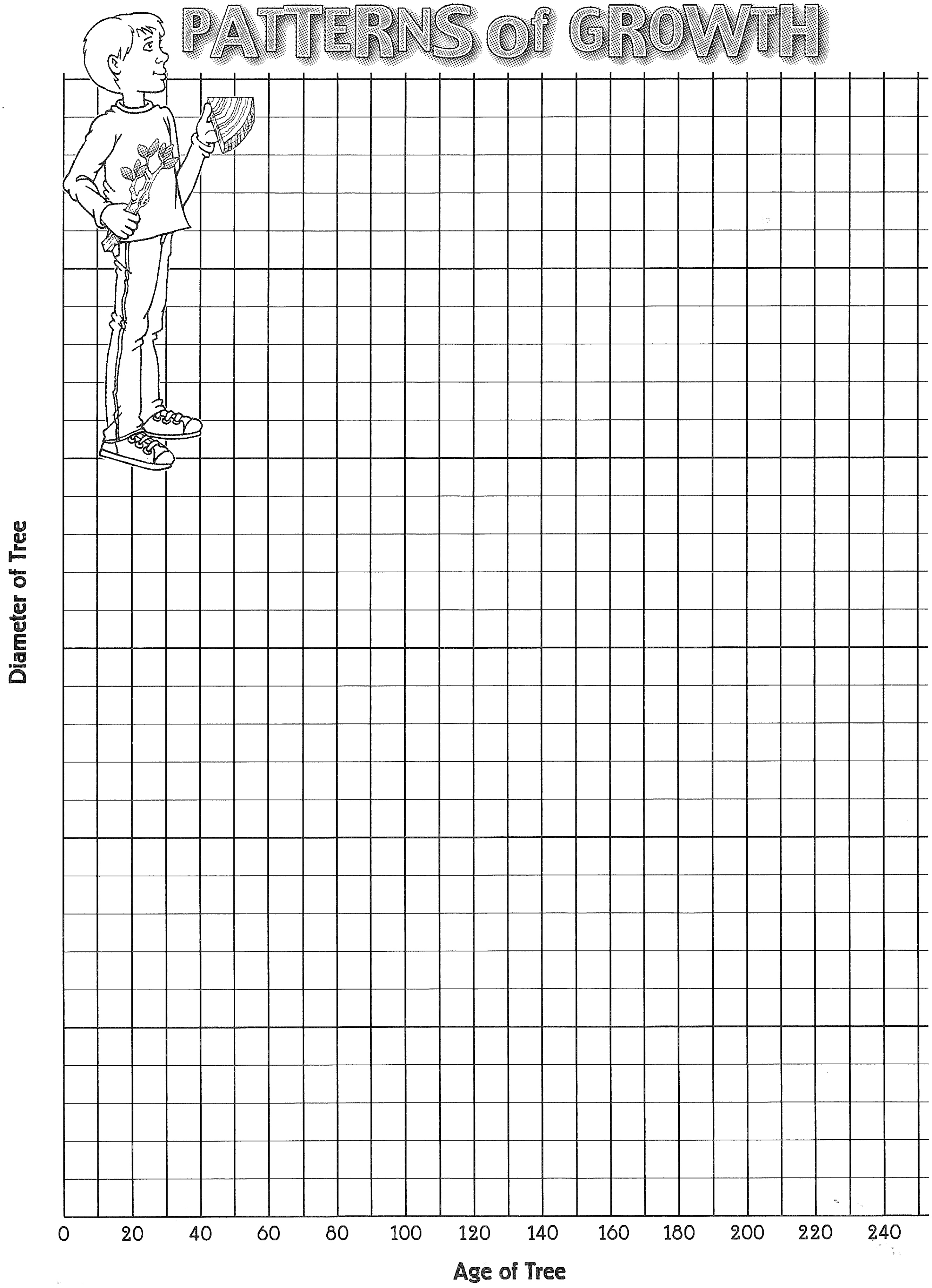
	HEIGHT		DBH*	
Age	Meters	Feet	Centimeters	Inches
20	18.6	61	25	9.8
30	26.5	87	36	14.2
40	31.4	103	43	17
50	35.1	115	48	19
60	37.5	123	52	20.4
70	39.6	130	55	21.6
80	41.1	135	57	22.4
90	42.1	138	59	23.1
100	43.3	142	60	23.7

• DBH = Diameter at Breast Height (4.5 ft or 1.37 m)

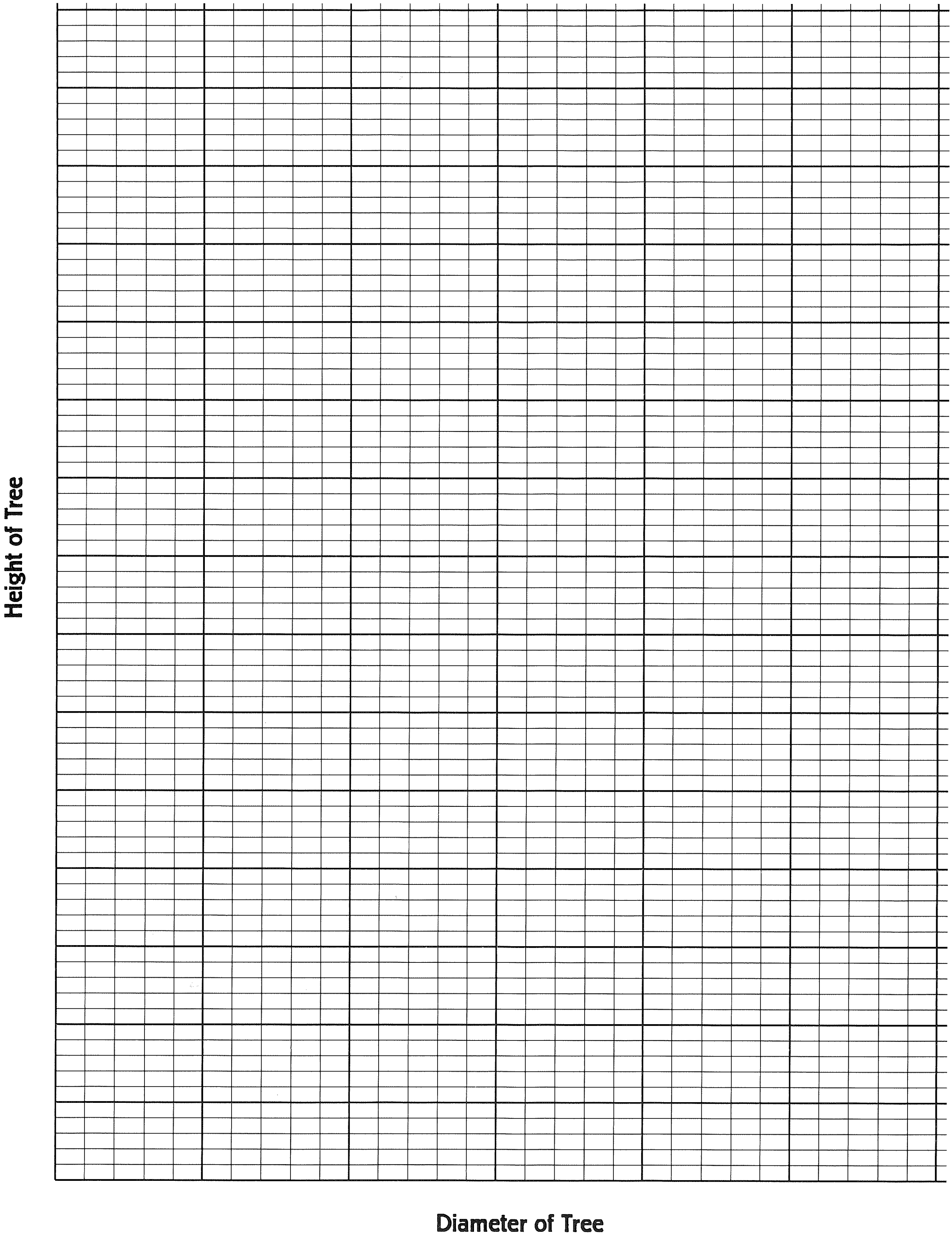
PATTERNS of GROWTH



PATTERNS of GROWTH



PATTERNS of GROWTH



The Teachable Moment

by Suzy Gazlay

Trees

How much weight can you lift? Imagine using a rope to haul something very heavy—like a big bucket of water—from the ground to the roof of a two-story building. What if you had to do this over and over in a single day until you had raised more than one thousand pounds of water to the roof? Or what if, rather than lifting all that water, you had to suck it up through a very long straw? And what if you repeated this chore daily throughout the spring and summer?

Such a task may sound impossible for a person to accomplish, but it's normal routine for a tree. Each day during its growing season, the tree moves a large quantity of water all the way from the tips of its roots to every single leaf and all the living parts of the tree between. The hotter the weather, the more water is moved. Of course, the amount of water transported also depends upon the size of the tree. On a hot summer day, a tall tree might move more than 200 gallons. Since a gallon of water weighs eight pounds, the total amount of water transported in one day by that particular tree is 1600 pounds!

Trees accomplish this feat with a system more similar to using a bunch of straws than lifting buckets. Just inside the bark is a living part of the tree known as the *sapwood*. In the middle region of the sapwood is a layer called the *cambium*. The cells of the cambium keep dividing to create new cells on either side of it. These cells line up to become the tree's transportation structures, *xylem* and *phloem*. The xylem, located next to the cambium on the inner side, starts out as strings of cells with hard outer walls which form straw-like tubes. Water and minerals are transported via these tubes from the roots to the rest of the tree. The annual rings we can see when a tree is cut down are the new xylem cells that were formed each year. Phloem, the tubes on the outer side of the cambium, consists of rows of living cells touching each other. The cells are long and have many tiny holes in their walls. Food in the form of a syrupy solution (*sap*) made by the leaves is passed along the phlöem, moving through the holes from cell to cell, and reaching every living part of the tree with nutrients for growth or energy.

Individual phloem and xylem tubes are really tiny: one five-hundredth of an inch wide or even smaller. They are also incredibly strong. A tall column of water is quite heavy in proportion to the size of the tube enclosing it. The walls of the tube must be very tough to keep from bursting under that much pressure. As the tree grows, a hard woody substance called *lignin* builds up inside the older xylem nearest the center of the tree. Water and food can no longer get into these cells, and they die, but the strong cell walls remain. It is the strength of the walls of the xylem that makes wood so strong. The dead xylem in the center of the tree forms the *heartwood*, serving the important function of supporting the tree.

As the xylem moves water up into the tree, the tubes operate very much like a bunch of drinking straws. When you suck on a straw, you cause the air pressure to decrease around your end of it. At the same time, the weight of the atmosphere is constantly pushing down on the liquid you are drinking, so when you lower the pressure at one end, the liquid is pushed up the straw and into your mouth. In a similar fashion, low pressure at the upper end of the xylem "straws" results in water moving up the tubes. A process called transpiration causes the drop in pressure. Water inside the leaves (needles) evaporates through tiny holes called *stomates*, located on the underside of the leaves or in the cuticle of the needles. As the water departs, the inside of each leaf (needle) dries out, causing its internal pressure to drop, and water moves up the xylem tube to replace what was lost. At the lower end of the tube, water leaving the root tips creates another area of low pressure, so more water is pulled in from the surrounding soil.

Several properties of water help this process along. The combined effects of cohesion and tensile strength enable the water molecules to stick together even as they are being stretched out inside the tube. Thus the water is drawn along like an endless rope, constantly being replenished with additional water from the roots. In addition, capillary action causes the water to pull up along the sides of the tube, so the water is actually helping to move itself along.

Bark consists of two layers, the *inner bark* and the *outer bark*. The inner bark contains the *bark cambium*. Like the inner cambium, this thin layer of living cells divides and forms new cells on either side of it. Those on the inside become food storage cells and additional phloem. Those on the outside become part of the *bark*. Eventually the walls of the new bark cells fill with a waxy substance and die. These dead cells become the protective layer of the outer bark. As the tree grows outward, the bark will stretch and sometimes tear, but new bark cells are continually being formed to keep the inner tissues protected.

The primary job of the bark is to protect the phloem and xylem directly beneath it from weather, disease, loss of water, and damage caused by fires, insects, and other threats. If a tree's bark is stripped off or cut deeply all the way around, water and nutrients will no longer be able to flow, and the tree will die.

Interesting Facts

- Only one percent of the water transported within a tree is actually used to make food in the process of photosynthesis. The other 99% evaporates from the leaves (needles). Its function is to keep additional water moving up the tree.
- Conifers can live in areas too dry for other trees because their needles are stiff and waxy, conserving water better than broad leaves.
- The structure of a shrub or bush is similar to that a tree, but on a smaller scale.
- Other green plants also have vascular systems containing a cambium, phloem, and xylem. However, these structures are arranged in groups called *vascular bundles* instead of in rings.
- The amount of lignin in the xylem cells determines the hardness of a tree's wood. The more lignin present, the thicker the cell walls and the harder the wood.

- Mistletoe is a parasitic plant that lives off the sap of a tree. Its roots grow into the trunk of the tree to reach and draw out the sap.
- On the average, a tree growing in a temperate climate gets about an inch thicker each year.
- Not all trees are made of wood. One example of such a tree is the palm tree. Its "trunk" actually consists of the bases of former leaves tightly packed together.
- The lightest type of wood in commercial use is balsa. It weighs as little as three pounds per cubic foot, which is one-third the weight of cork.
- The bark of a giant sequoia can be more than a foot thick. The beech tree's bark is less than one-fourth of an inch thick.
- Annual rings show that a tree makes wood only during the warmer part of each year. Not all trees have annual rings.
- A large oak tree may draw over a thousand gallons of water a day.
- Bark that can stretch as it grows stays smooth on the tree. Bark that is rough and cracked isn't as stretchy.
- Early settlers blazed trails and cleared fields by *girdling* the trees they didn't want. It was easier and faster to make a deep cut all the way around the tree and wait for it to die than it was to chop it down.
- The bark of a tree doesn't usually get much thicker each year because some bark is lost from the outside as new bark is added from within.
- Most paper is originally made from pine trees. An average-sized pine tree—one foot in diameter and 60 feet tall—provides the equivalent of about 80,500 sheets of standard sized copy paper.

Things to Do

- Examine the stump of a tree, a log, or a "tree cookie" (crosswise slice of the trunk or branch).

  Find and identify the heartwood, sapwood, bark, and annual rings. Each ring has two sections. The lighter, softer part is the wood that grew during the spring, and the darker, harder part is the wood that grew during the summer. Determine the age of the tree or branch from which this sample was taken.

  Depending upon the sample, you may be able to see the phloem, a very thin layer squeezed between the bark and the xylem.

  Compare the width of the rings. Narrow rings indicate a year when the tree did not get as much water and/or food, perhaps because it was shaded. Rings that are wider on one side of the center than on the other indicate that the tree was leaning in the direction of the wider part. Look for lines (not splits) moving from the center towards the bark. These are *medullary rays* that transported food and water across the trunk. The splits in the wood occurred as the stump or "cookie" dried out.

  Locate places where the ring seems to dip towards the center in a V shape. These spots show where branches once grew when the tree was smaller.

  Look for the bark cambium on the inside of the outer bark.
- Collect samples or "cookies" of different types of wood and/or bark. Identify them as much as possible and compare the various characteristics. Compare the samples to products made from that type of wood. [pine boxes, cedar pencils, furniture, etc.] Caution: do not injure the trees in the process of gathering samples!
- Compare the weight of the water used by different trees to your own weight. (Amounts for a large oak and a "typical large tree" can be found in this article.) How many of you would it take to weigh that much? Try to find the amount of water used by other trees.
- Several universities and organizations have extensive information on the Internet both about trees in general and about specific types of trees. Use this information to learn more about trees native to your area or trees from other locations.
- Do the "Tree Theatrics" found in this issue of *AIMS®*.

Resources

For more information about the growth of trees, please see "Patterns of Tree Growth" in *AIMS* ® Vol. XIII, No.1.

Aronson, Steven M. L. *Trees: Trees Identified by Leaf, Bark, & Seed.* Workman Publishing Company. New York. 1998.

Cassie, Brian. *Trees (National Audubon Society First Field Guide).* Scholastic, Inc. New York. 1999.

Gamlin, Linda. *Eyewitness Explores: Trees.* Dorling Kindersley Publishers. New York. 1997.

Ganeri, Anita. *What's Inside Plants?* Peter Bedrick Books. New York. 1993.

Literature Connections

For Younger Readers:

Brenner, Barbara, and May Garelick. *The Tremendous Tree Book* (Reading Rainbow Book). Caroline House. Honesdale, PA. 1992.

Dorros, Arthur. *A Tree is Growing.* Scholastic, Inc. New York. 1997.

Fowler, Allan. *It Could Still Be a Tree.* Children's Press. Chicago. 1990.

Morrison, Gordon. *Oak Tree.* Houghton Mifflin Co. Boston. 2000.

For All Ages:

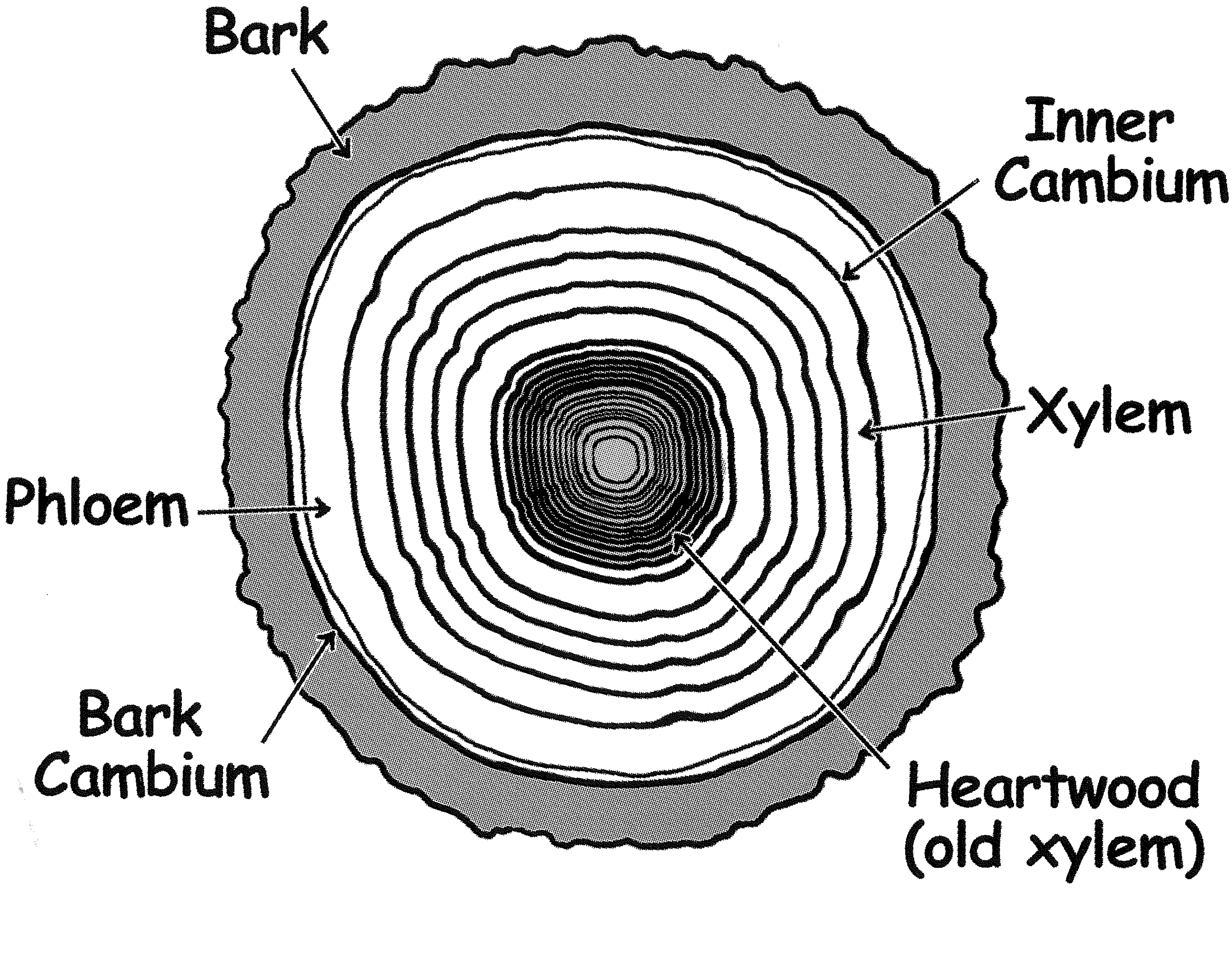
George, Kristine O'Connell. *Tree Poems.* Houghton Mifflin Co. Boston. 1998.

Internet Connections

http://forestry.about.com/cs/treeid/index.htm
Follow links for extensive information about the most common North American Trees and other forestry topics

http://www.rbnc.org/maplesyr.htm
Information about "maple syruping" highlighting the inner structure and workings of a maple tree

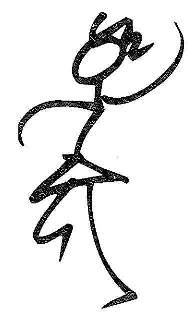
Cross Section of a Tree



Tree Theatrics*

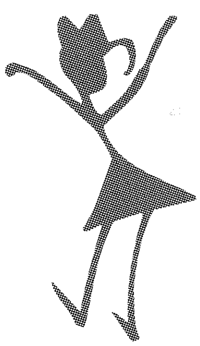
*adapted by Suzy Gazlay

A tree is a living organism that needs food and water for energy and growth, as well as stability and some sort of protection. It would be difficult to watch the actual processes at work providing for the tree's needs, but we see their effects in a healthy, growing tree—or, if they are damaged or disrupted, in a dying or dead tree.



Tree Theatrics provides an effective way to illustrate the inner and outer workings of a tree. Each participant plays the role of a specific tree part. A minimum of 21 actors is needed, but it is even more effective with a larger group—perhaps combining classes. Suggestions are given for sounds and motions to use for each layer, but substitutions may certainly be made.

Build the tree from the inside out, adding one system at a time. You may wish to add information about the "job" of each system as you build the tree (see *The Teachable Moment: Trees)*. For increased dramatic effect, have those students already in place repeat their sounds and motions, beginning with the heartwood and building as you move outward, each time a new group is added.



The **Heartwood** (one student) supports the tree, standing tall and strong. The student portraying the heartwood should plant his/her feet firmly and reach up as if supporting a heavy load.

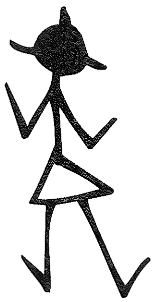
Motion: flexing muscles. Sound: "I'm strong! I'm strong!"

The **Taproot** (one student) reaches down and anchors the tree firmly into the ground. He/she lies on the ground with feet touching the feet of the heartwood, arms stretched back beyond his/her head, pretending to reach far down into the ground.

Motion: reaching; digging motions with fingers. Sound: "Reach! Reach!"

The **Lateral Roots** (two students, with long hair if possible) represent all the hundreds of tiny roots that draw in water and nutrients from the surrounding soil. These students lie down with feet touching the taproot. Spread out their hair for added effect.

Motion: clench and unclench fingers. Sound: "Draw! Draw!"



Unless otherwise noted, the number of students in each of the following roles depends upon the total number of students left. If there are enough students, each group should have a few more actors than the previous group.

The **Xylem** carries water and nutrients from the roots up the trunk and throughout the tree, all the way to each leaf. Students portraying the xylem form a circle around the heartwood, facing inward and being careful not to step on the roots.

Motion: dip down, touch the ground with arms outstretched and stand up as if lifting until upright with hands reaching over head; keep repeating. Sound: loud and long slurp as they come up.

The **Inner Cambium** is a sort of "mother" layer, creating new cells for the layers on either side of it. These students stand in a circle with arms stretched out to the side as if reaching for a big hug, alternating facing in and out.

Motion: leaning forward as if to hug. Sound: "Hmmm! Hmmm!"

The **Phloem** carries the food made by the leaves and distributes it throughout the tree.

Motion: stand on tiptoes and reach high, then swoop to the ground.

Sound: "Blblblblblblblbl!"

The **Branch** (one student) stands sideways with one arm touching the inner cambium and the other reaching out as far as possible.

Motion: lean into the reach. Sound: "Reach! Grow!"

The **Leaves** (two students) use energy from the sun to make food to feed all the living parts of the tree. Have the two students stand on either side of the branch, touching the extended arm.

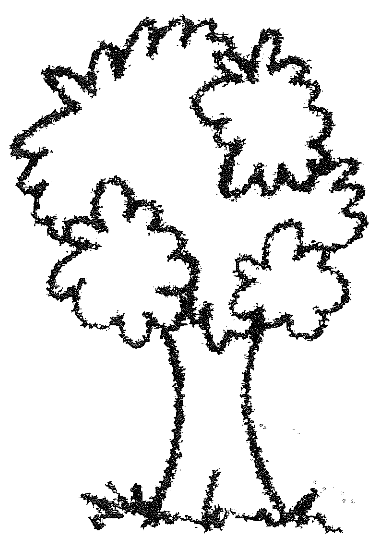
Motion: flutter the fingers. Sound: "Energy! Food!"

The **Bark Cambium**, like the inner cambium, produces new cells for the systems on either side of it. Follow the directions for the inner cambium.

Motion: leaning forward as if to hug. Sound: "Hmmm! Hmmm!"

The **Outer Bark** protects the tree from weather, animals, fire and other potential damages. Have the remaining students form a circle, facing outward.

Motion: menacing pose. Sound: "Bark! Bark!" (may also add "Ruff!")



* Variations of this activity have circulated among teachers for many years, evolving to reflect new ideas and suggestions. It may well continue to change and grow as creative teachers adapt it for their students. Although the author was able to locate a few versions in written form, so far the original source is not known.

Leaf the Counting to Me

by David Mitchell

Topic
Problem Solving and Estimation

Key Question
How many leaves are on a tree?

Learning Goals
Students will:
1. make an initial estimate of the number of leaves on a tree, and
2. make a strategic estimate of the number of leaves on the same tree.

Guiding Documents
Project 2061 Benchmark
- *Scientific investigations may take many different forms, including observing what things are like or what is happening somewhere, collecting specimens for analysis, and doing experiments.*

NRC Standards
- *Different kinds of questions suggest different kinds of scientific investigations. Some investigations involve observing and describing objects, organisms, or events; some involve collecting specimens; some involve experiments; some involve seeking more information; some involve discovery of new objects and phenomena; and some involve making models.*

*NCTM Standards 2000**
- *Compute fluently and make reasonable estimates*
- *Apply and adapt a variety of appropriate strategies to solve problems*
- *Analyze and evaluate mathematical thinking and strategies of others*
- *Create and use representations to organize, record, and communicate mathematical ideas*

Math
Problem solving
  estimation

Science
Life science
  botany

Integrated Processes
Observing
Comparing and contrasting
Communicating
Collecting and recording data

Materials
For each group:
  tree (see *Management 1*)
  calculators

Background Information
This activity is designed to allow students an opportunity to use estimation in the context of a census. Students are asked to estimate the number of leaves on a tree. Students will need to develop a strategy to conduct a count on what on the surface seems to be uncountable. Students will compare different strategies and discuss the advantages and disadvantages of each.

Management
1. Select a tree or trees that will present a challenge for the students but not overwhelm them. Select a broad leaf tree that is taller than the students are but shorter than a one-story building.
2. Focus the students on the process of making a good estimate, not the correct answer.
3. Have students work in groups of three or four.

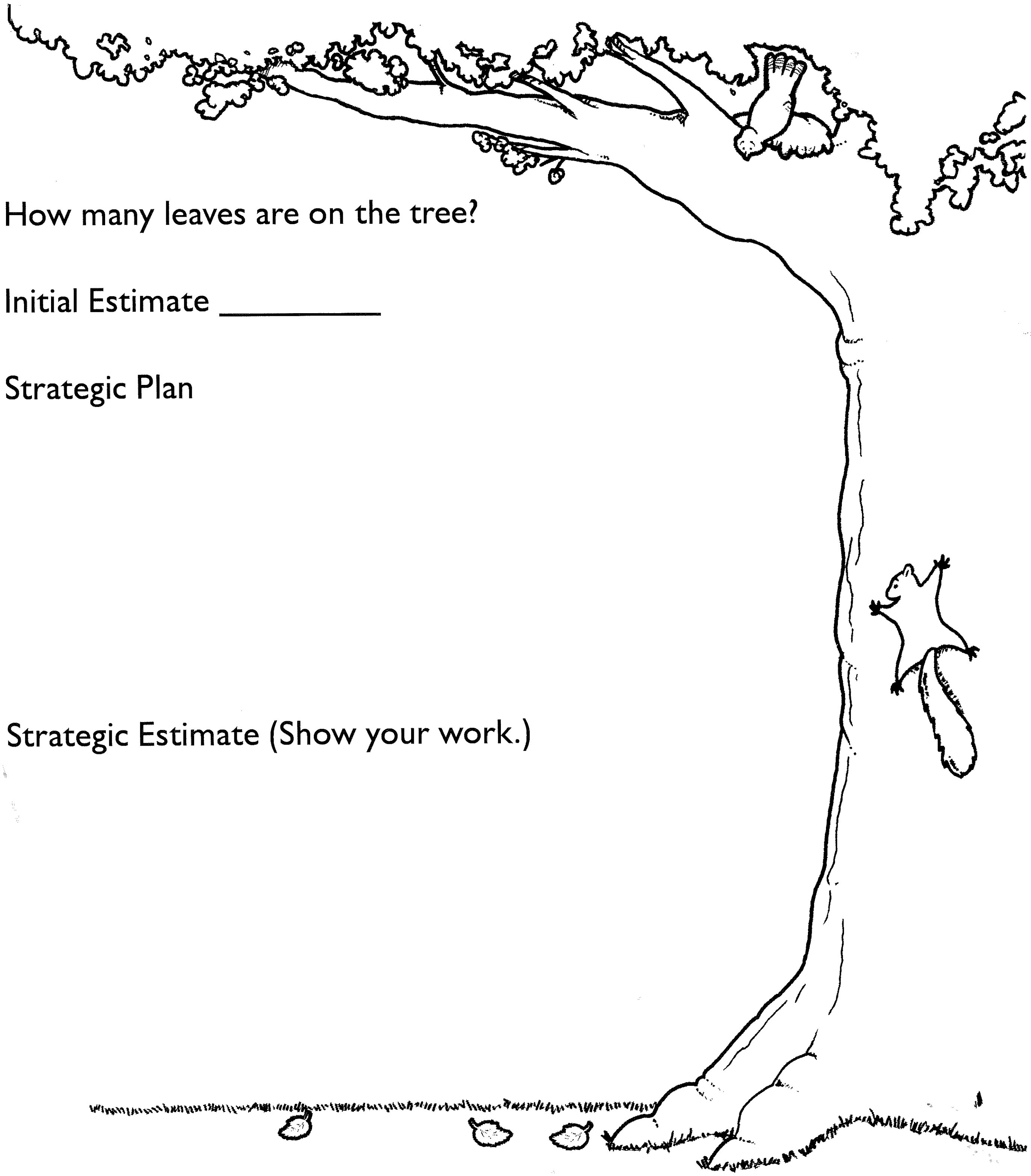
Procedure
1. Ask the *Key Question* and state the *Learning Goals.*
2. Direct the students to look at the tree/trees you have previously selected. Give them three to four minutes to come up with an estimation of the number of leaves they think are on the tree.
3. Ask the students to record their initial estimates. Discuss with the students the range of the estimates. Ask them how they arrived at their estimates.
4. Challenge each student group to plan a more strategic estimate on the number of leaves on the tree. Tell them they must develop a written plan on how they are going to conduct the count.
5. Review the student plans, then have each group conduct the count and record their strategic estimate. Have them compare their initial estimates to their strategic estimates.
6. Initiate a class discussion on how each group solved the problem.

Reflecting on Learning
1. Why do you think you may find a difference between your initial estimate and your strategic estimate?
2. What other types of counts use strategic estimates?
3. How could you go about estimating the number of leaves on a larger tree?
4. What plan do you think produced the best estimate for the number of leaves on the tree?
5. If you could now change any part of your groups' plan, what would it be and why would you change it?
6. What math skills did you use to solve this problem?
7. Why isn't it practical to actually count every leaf on the tree?

Evidence of Learning
1. Listen for student talk during the activity as well as responses to the *Reflecting on Learning* questions.
2. Look for clarity in completing the written plan.

* Reprinted with permission from *Principles and Standards for School Mathematics,* 2000 by the National Council of Teachers of Mathematics. All rights reserved.

Leaf the Counting to Me



How many leaves are on the tree?

Initial Estimate __________

Strategic Plan

Strategic Estimate (Show your work.)

Symmetry

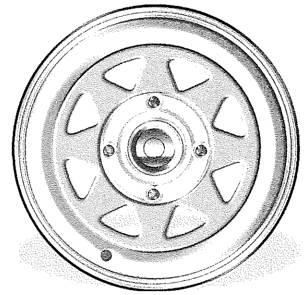
by Richard Thiessen

Wheel-World Symmetry

Car wheels are an excellent place to look in the real world for rotation symmetry and line symmetry. Until seven or eight years ago, car wheels were generally very plain looking things that were likely painted black or the same color as the car. These plain wheels were then covered with chrome wheel covers or hubcaps to dress them up. Today there are very few new cars that actually have wheel covers, but rather the plain painted wheels have been replaced with shiny chrome wheels that don't need wheel covers. If you are not satisfied with the wheels on you car, I've discovered that nearly every tire store in this area has an almost limitless variety of fancy chrome wheels either in stock or available through a catalog.

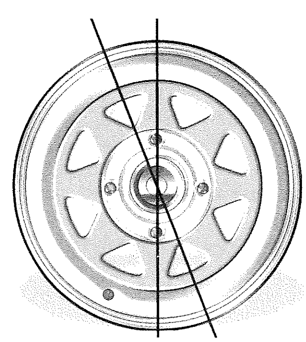
For the past several months as I have focused on and written about symmetry, looking at the wheels of cars has become something of an obsession. As I wait at an intersection for the light to change, I check out the wheels on nearby cars. I can't walk across a parking lot without noticing the wheels of the cars. How many facets or spokes do they have? Does the wheel have only rotation symmetry, or does it have line symmetry as well? The variety is nothing short of amazing. Fourteen different wheels are pictured throughout this article and on the two pages that accompany it. These were chosen from literally hundreds of pictures of wheels that I have collected over the past several months.

As those of you who regularly read *AIMS®* will know, this article is one in a series of articles that has focused on symmetry. The wheels that are pictured will provide the basis for taking one more look at symmetry, especially rotation symmetry.



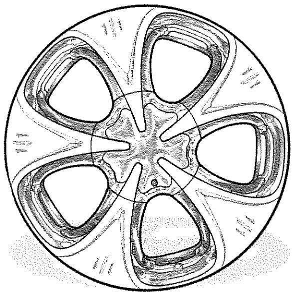
We start with a very simple design, an eight-spoke wheel. As you look for the symmetry of this wheel and the others that follow, disregard the arrangements of such things as lug nuts, valve stems, and also the company logos that may appear at the center of the wheel. Recall that an object has rotation symmetry if when it is rotated about a point or axis, it appears in the same position two or more times as it is rotated through 360 degrees. We can illustrate this by making a transparency of this page, cutting out the wheel, and placing it over the wheel as it appears on this page. Using a pushpin to align the centers of the two copies of the wheel, we can easily rotate the transparency copy to see that as it is rotated, it repeatedly appears in the same position. In fact this wheel will appear in the same position eight times in one complete rotation. Through how many degrees does the wheel rotate each time before it appears in the same position? One way to answer that question is to simply note that a complete rotation of the wheel is 360 degrees, so one-eighth of a complete rotation is 45 degrees. In other words, every time the wheel is rotated 45 degrees it appears in the same position. Another more direct way would be to place the copies of the wheel on a protractor. Then as the wheel is rotated we can determine the number of degrees of rotation by reading from the protractor. This can be done more accurately if the wheel has a couple of strategically placed marks to help align it with the protractor. Clearly, there is value in having students find the answer both ways. Using the protractor, they find it by measuring, and finding the answer by dividing 360 by eight (they are in essence finding it theoretically,) they are using a formula.

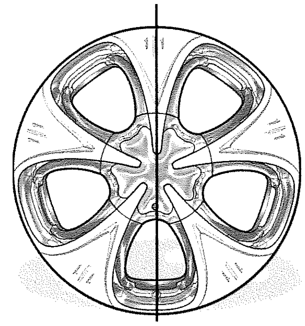
Continuing with the eight-spoke wheel we might ask, does this wheel have any lines of symmetry? Clearly the answer is that it does have line symmetry; in fact, it has eight lines of symmetry. Four of them are located at the centerline of the spokes and the other four pass between the spokes.



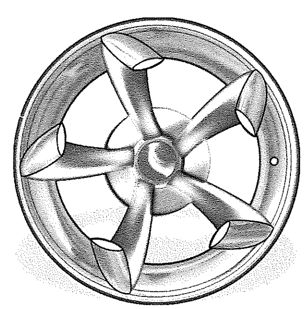
Now consider a five-spoke wheel. What can we say about the rotation symmetry of this wheel? We could again make copies of

the wheel and rotate the transparent copy to see that it will assume the same position five times in a complete rotation of the wheel. As before we can determine the number of degrees through which it rotates by simply dividing 360 by five. Or we can place copies of the wheel on the protractor and rotate the wheel until it assumes the same position, and then read the number of degrees of rotation. In any case, we find that the five-spoke wheel has 72-degree or five-fold rotation symmetry.



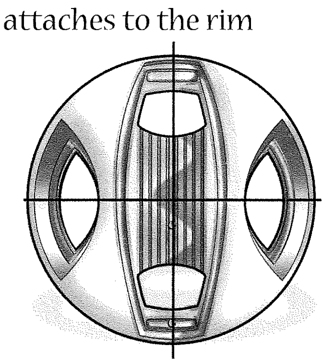


In addition to rotation symmetry, this wheel has five lines of symmetry, where each line passes through the center of one spoke and between two others. Note how the lines of symmetry for this wheel are different from those for the eight-spoke wheel. This will turn out to be an even and odd distinction. While wheels may or may not have line symmetry, whenever they do have this kind symmetry, those with an even number of spokes will have half that pass through the spokes and half that pass between them. While all lines of symmetry for wheels with an odd number of spokes will pass through one spoke and between two others just as was the case for the five-spoke wheel.

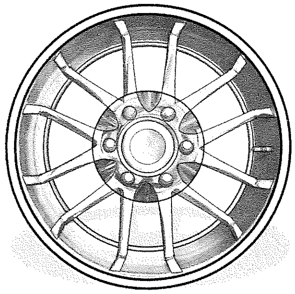


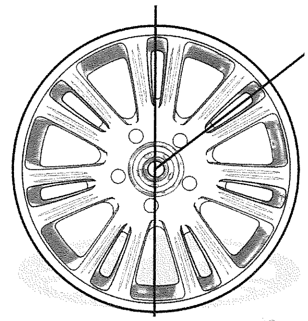
We said that wheels may or may not have line symmetry. The five-spoke wheel pictured above is an example of one that has no line symmetry. While it has five-fold rotation symmetry just as the previous wheel, the hook effect where each spoke attaches to the rim prevents it from having line symmetry.

A wheel that I found especially interesting has two-fold rotation symmetry and has two lines of symmetry that are perpendicular to each other. I'm sure you would find that the effect of this wheel is quite spectacular, especially if the car is passing slowly in front of you.

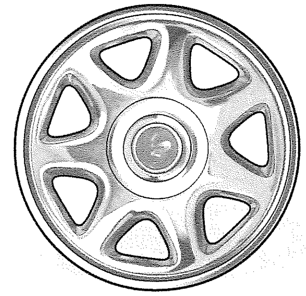


Another interesting wheel with six-fold rotation symmetry actually has 12 spokes, but they are arranged in pairs, thus preventing the rotation symmetry from being 12-fold. The wheel also has six lines of symmetry.

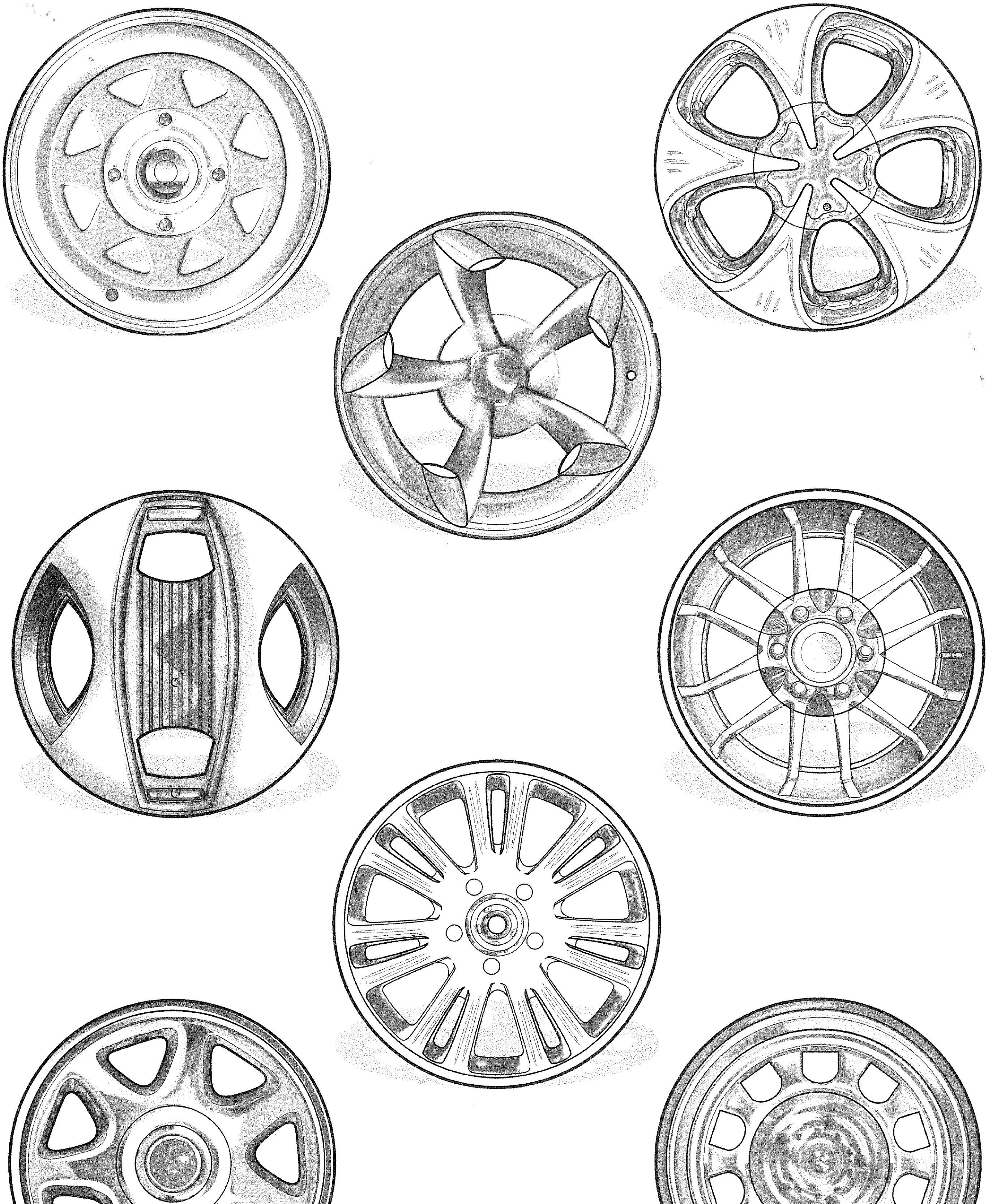


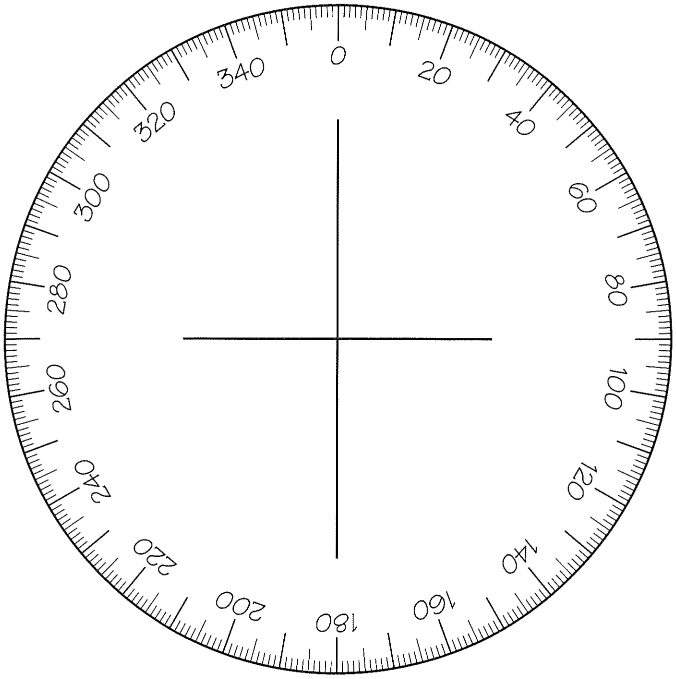


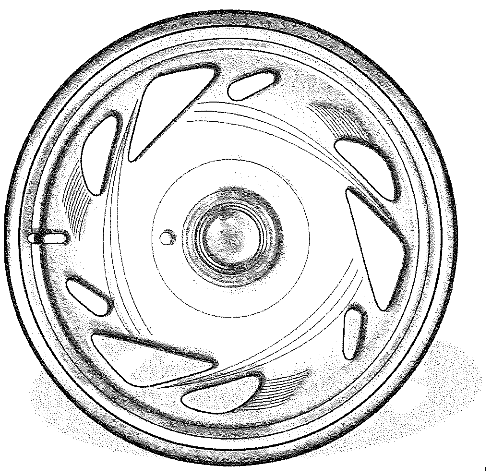
While I expect to see wheels with five, six, or eight spokes, I am always surprised to see one with seven-fold symmetry (pictured below); in fact, I have occasionally come across wheels with nine- and 11-fold rotation symmetry. While the angle of rotation for the wheel with nine-fold symmetry is 40 degrees, the angles of rotation for the other two wheels will be about 51.4 degrees and 32.7 degrees, respectively. If students check the degrees of rotation for the wheel with seven-fold symmetry with the protractor, they will probably find that it is close to 51 degrees. However, if they simply divide 360 by seven they will find the more precise answer.

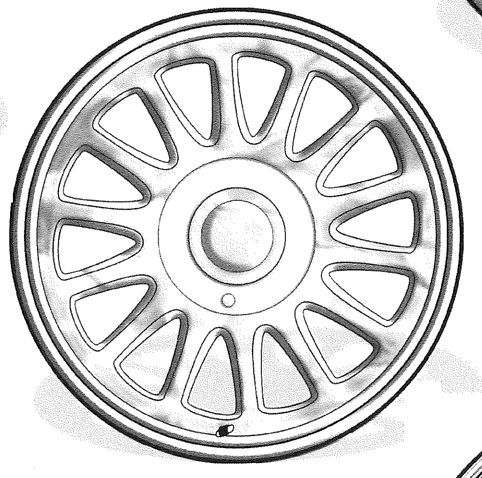


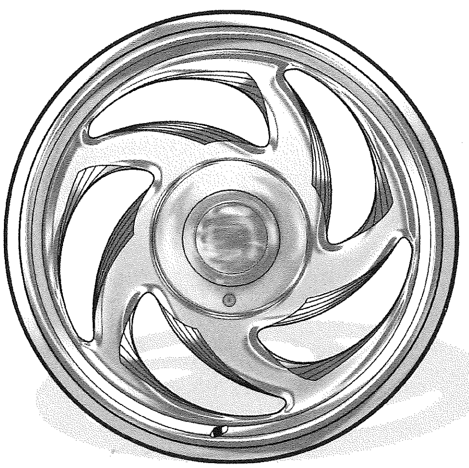
The attached pages have pictures of 14 wheels, including the ones we have looked at in the article. Also included is a circular protractor. I believe that an activity where students examine the various wheels and determine the kinds of symmetry they exhibit can be a powerful way for them to come to understand the meaning of rotation symmetry and to notice how that symmetry differs when line symmetry is also present. I would suggest that you make at least two copies of the attached pages, one a paper copy and the other a transparency. The cutout wheels can then be placed on the protractor with the transparency copy on top and a pushpin holding them in place.

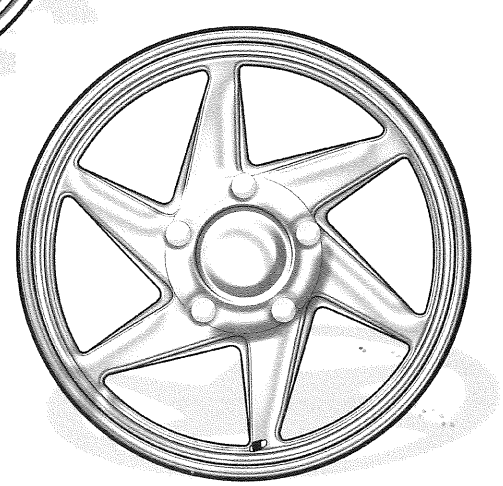



0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340









PRIMARILY PROBLEM SOLVING

SWEET SUMS

by Michelle Pauls

The NCTM Standards 2000 call for pre-K–2 students to *illustrate general principles and properties of operations, such as commutativity, using specific numbers*, and *model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.** This activity will address both of these standards as students use manipulatives to model addition problems as well as explore the various combinations of numbers that will produce the same sum.

This activity is divided into three sections. In the first section, students will use manipulatives to explore various ways in which objects can be arranged between two spaces to total a particular sum. In the second section, they will make a record of one possibility for each sum, and in the third section, they will find all possible ways to make each sum and record those solutions abstractly. As written, the activity asks students to deal with numbers from zero to six. You may wish to increase or decrease this range to make it more appropriate for your students.

In order to do this activity, each student will need six heart candies or other small manipulatives. Begin by handing out the first student page and candies to each student. Ask students to put candies on the pictures of the bags so that there are a total of three candies. (Be sure to remind them that bags may be empty.) Have several students share their combinations. Compare the various ways that candies can be placed in the bags to total three (0 + 3, 1 + 2). Repeat this process for several other numbers until students are comfortable with making sums.

Once this initial time of exploration is over, tell students that they are now going to record one way to make each number from one to six using candies divided between two bags. Hand out the second and third student sheets and colored pencils or crayons. Students should work individually to record one solution for each problem by drawing candies in the bags. If this is too abstract for your students, you can have them glue candies onto the student sheets or another paper to represent one version of each addition problem.

After all students have recorded their solutions, have them get together into small groups of four or five. Instruct students to share their solutions and to compare the different solutions discovered within the group. Tell students that their challenge is to discover and record *every* way that each sum can be reached by putting candies in two bags. Hand out a sheet of chart paper for each group to use as they record their solutions. Each solution should be recorded as a drawing showing the number of candies in each bag, as well as the total.

An important distinction to be aware of is the difference between combinations and permutations. A combination is a grouping of objects, numbers, etc. without regards to order. In this case, 1 + 2 = 3 would be seen as the same as 2 + 1 = 3. A permutation is a grouping of objects, numbers, etc. in which order matters. In this case, 1 + 2 = 3 would be seen as a different from 2 + 1 = 3. This activity is intended

to be a study of combinations, but you may wish to extend it to study permutations as well. If you want students to find all of the permutations of each solution, be sure that they are aware that the order of the numbers matters. Likewise, if you only want to study combinations, be sure that students are aware that order does not matter.

Once groups have had sufficient time to discover all of the solutions, hand out the final student sheet. Challenge each group to record their solutions in the numeric [abstract] form in the appropriate space on the student sheet. Come together for a final time of class discussion where groups share their solutions and compare with other groups. If there are differences, discuss why some groups got answers that others did not.

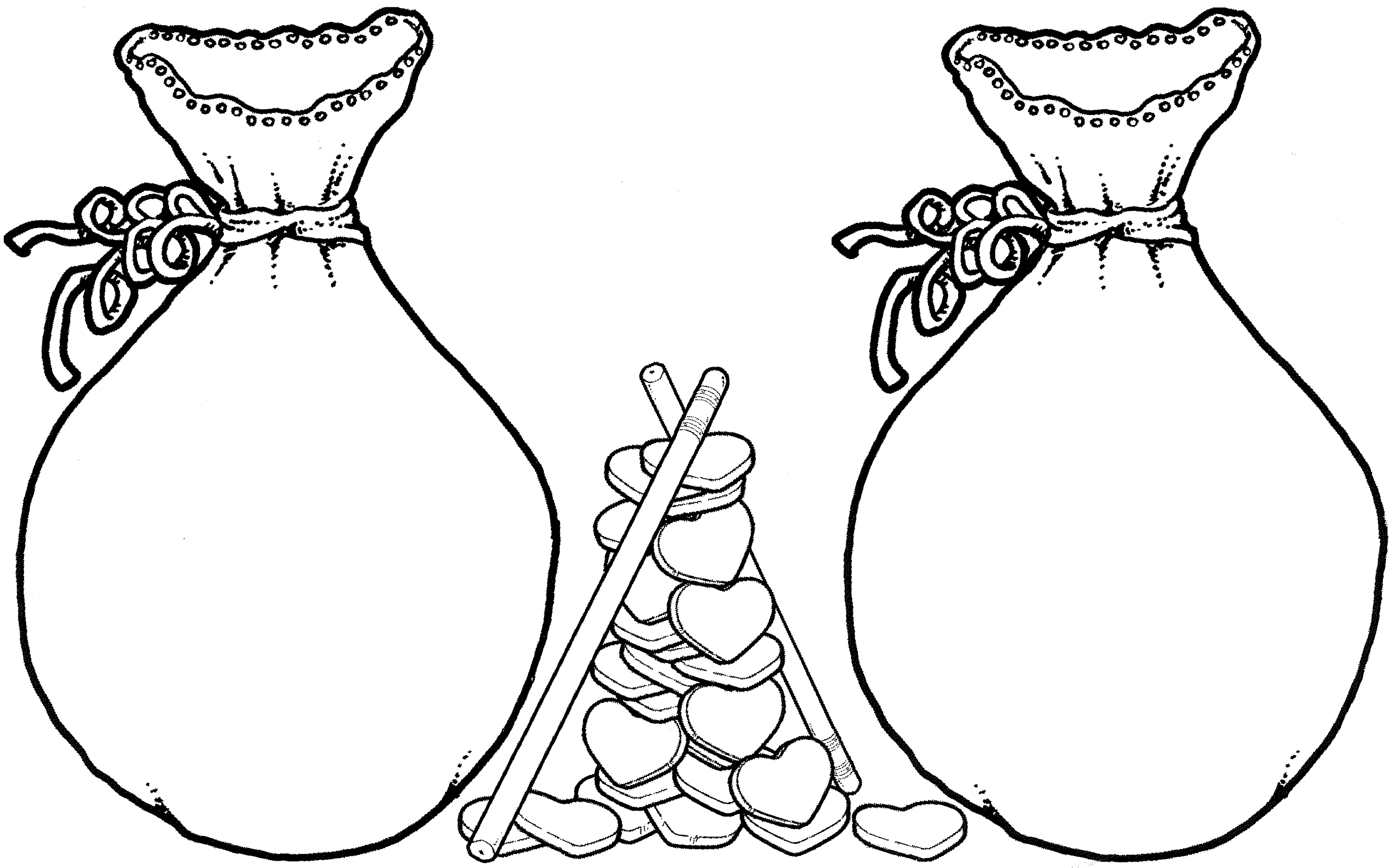
One extension for older students would be to look for patterns in the number of combinations possible for each sum. Zero and one both have one combination ($0 + 0 = 0$, $0 + 1 = 1$), two and three both have two combinations ($0 + 2 = 2$, $1 + 1 = 2$; $0 + 3 = 3$, $1 + 2 = 3$), etc. Another extension would be to look for patterns in the number of permutations possible for each sum. Zero has one ($0 + 0 = 0$), one has two ($0 + 1 = 1$, $1 + 0 = 1$), two has three ($0 + 2 = 2$, $2 + 0 = 2$, $1 + 1 = 2$), etc.

I hope you and your students enjoy this activity. If you have any questions or comments about this, or any other *Primarily Problem Solving* activity, feel free to email me (meyoungs@fresno.edu) or contact me here at AIMS. Stay tuned for the next installment of this column in the April magazine.

* Reprinted with permission from *Principles and Standards for School Mathematics,* 2000 by the National Council of Teachers of Mathematics. All rights reserved.

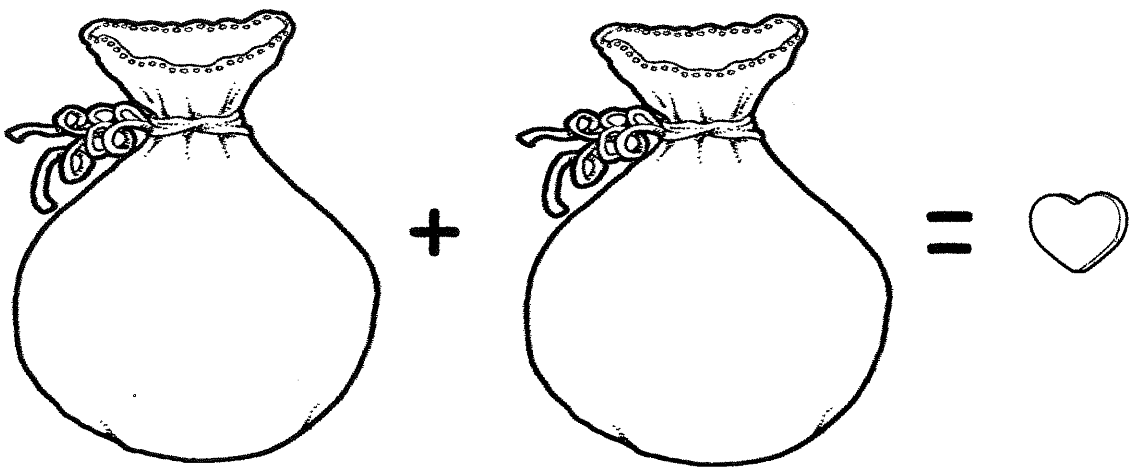
SWEET SUMS

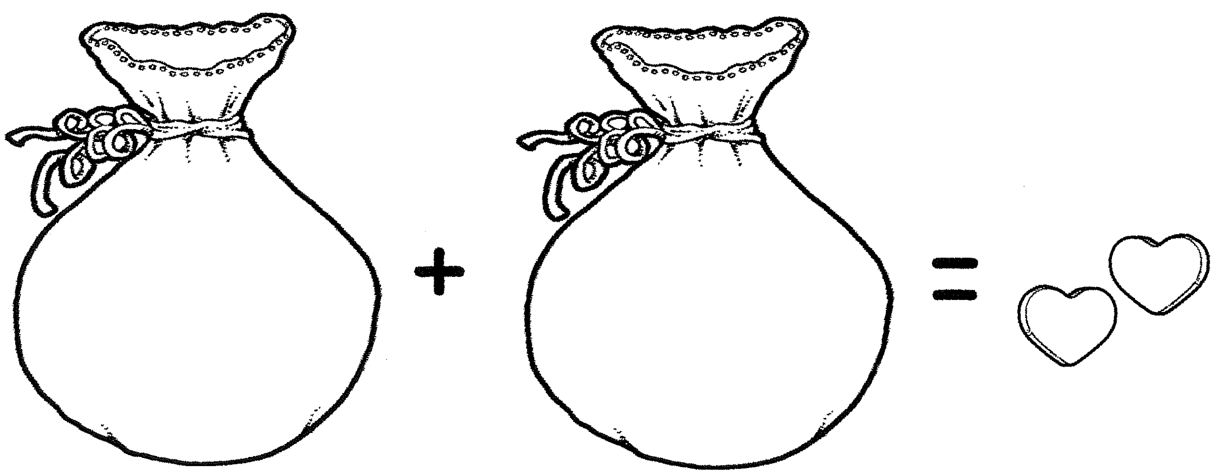
Place candies in the bags to equal the sums your teacher gives you.
(Bags may be empty.)

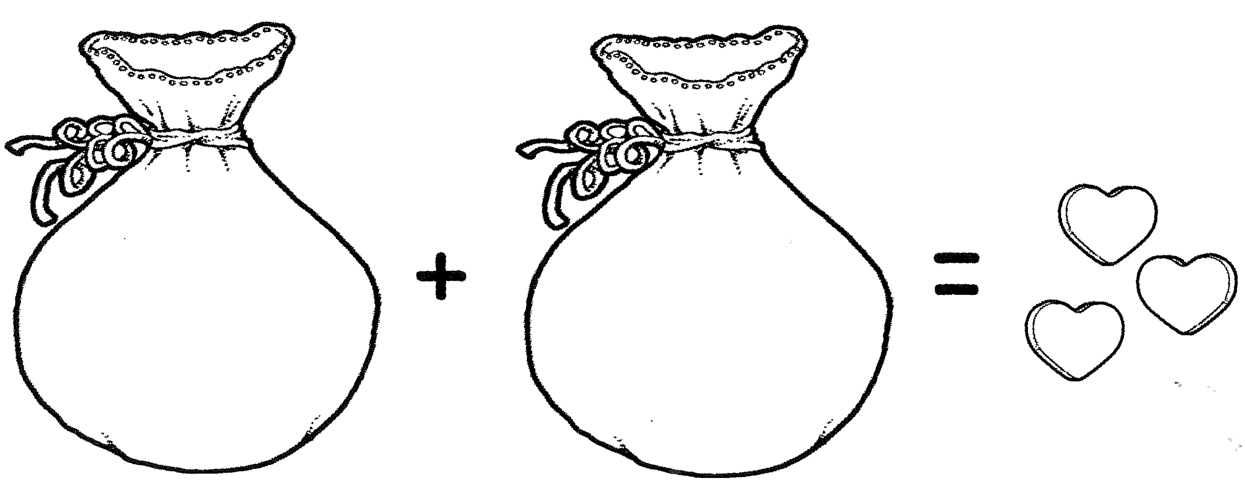


SWEET SUMS

Draw candies in the bags to make the number sentences true. (Bags may be empty.)

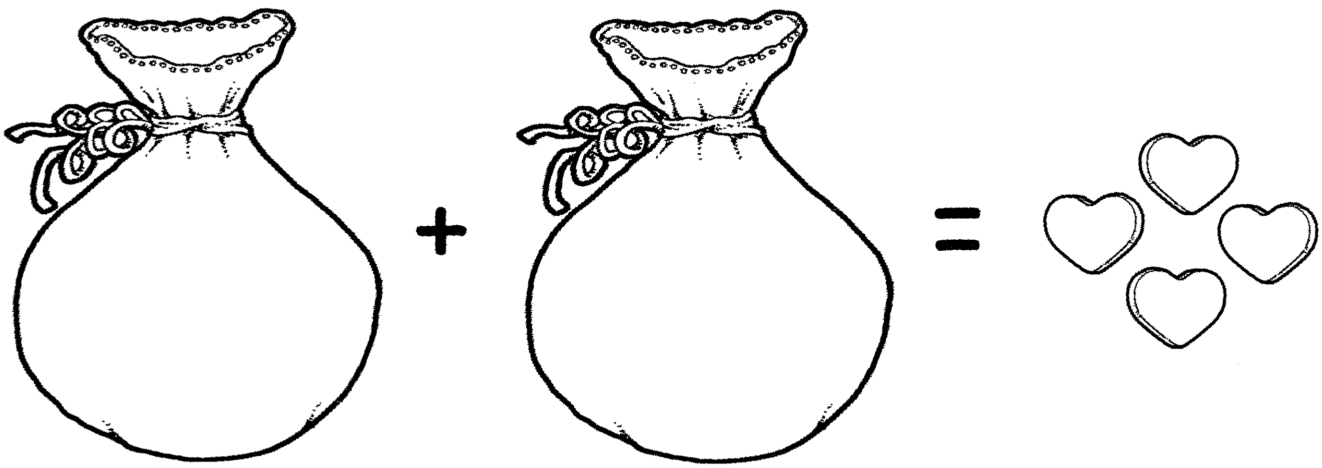


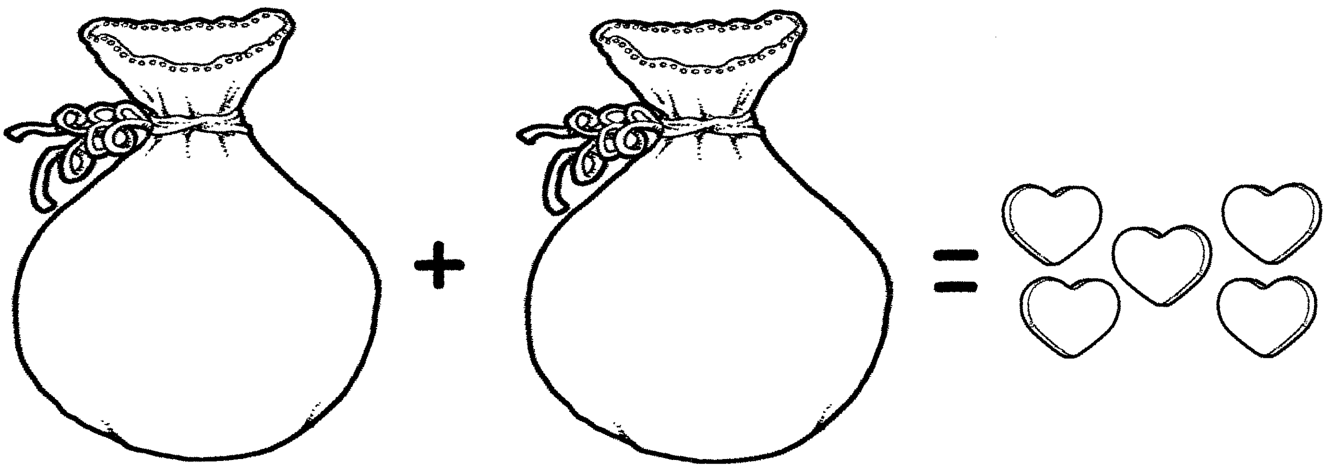


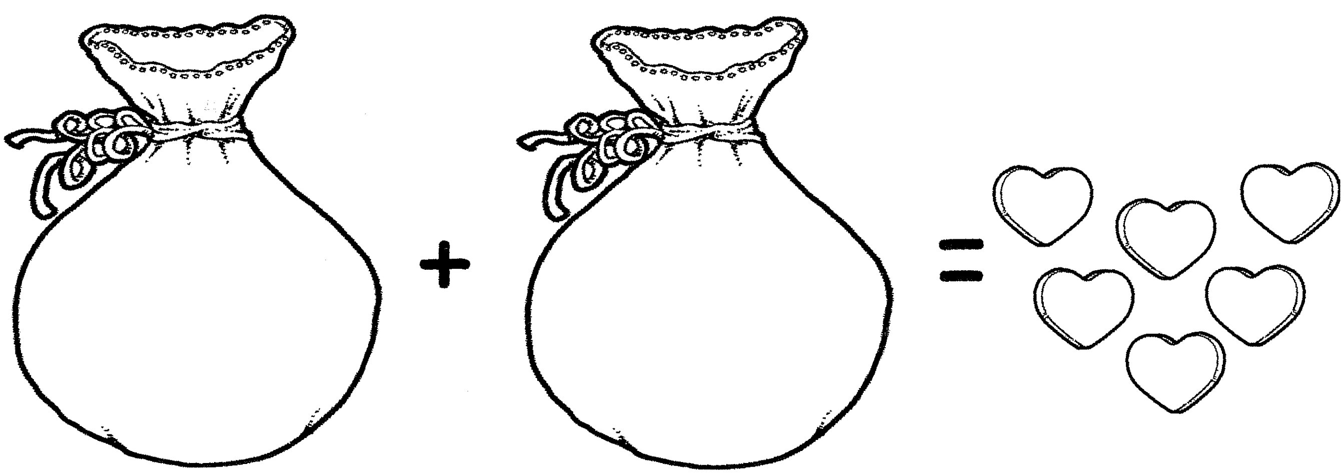


SWEET SUMS

Draw candies in the bags to make the number sentences true.
(Bags may be empty.)







SWEET SUMS

1. How many ways can you make six? Write each way.

2. How many ways can you make five? Write each way.

3. How many ways can you make four? Write each way.

4. How many ways can you make three? Write each way.

5. How many ways can you make two? Write each way.

6. How many ways can you make one? Write each way.

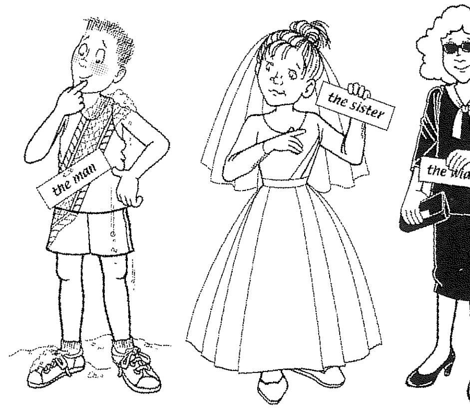
Puzzle Corner
Puzzling Problems
by Dave Youngs

Is it against the law for a man to marry his widow's sister? Trick questions like this one almost always pique our interest. We usually delight in these cleverly crafted riddles and brain teasers that catch us off guard. Yet once tricked, most of us are careful not to be tricked again. Anyone who has been snared by the above question is not likely to be fooled by it in the future. (If you have not encountered this question before, try to determine why it is a trick question.) This month's *Puzzle Corner* is a collection of trick math questions that will require careful reading and thinking on the part of your students, if they are to be answered correctly. Hopefully, students will take the same delight in these problems (once they discover that they are tricky) that they would in figuring out riddles or trick questions like the one above.

Some educators disparage trick questions like the ones included in this activity. They feel that they have no part in the mathematics classroom, calling them confusing and counterproductive for students. While I can understand why they might feel this way, I believe that the initial confusion can lead to something quite productive for students—the realization that these problems need to be read critically before they are answered. The higher-level thinking that this entails is quite valuable and well worth any initial frustration students may feel when working on problems of this sort.

When you introduce these problems to your class, you'll have to decide if you want to tell them ahead of time that these are trick questions or let them find this out on their own. For me, the latter option is preferable. Once students discover they have been tricked, they are not as likely to be tricked again and will read future questions more carefully.

I hope that you and your students enjoy these puzzling problems. The solutions and another puzzle will appear in the next issue of *AIMS*®. If you need the solutions before then, you can send me an email (dyoungs@fresno.edu) or write to me in care of AIMS.



PUZZLING PROBLEMS

Answer the following math problems. Be sure to read them carefully.

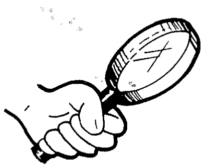
1. You have two coins that are worth 30 cents. One of the coins is not a nickel. What are the two coins?

2. Juan had 35 grapes in his lunch pail. He ate all but nine. How many did he have left?

3. Nicki has eight identical blue socks and eight identical green socks all mixed up in her sock drawer. If she reaches in without looking, what is the minimum number she must pull out to get a matching pair?

4. How much dirt is in a hole that is one meter deep, two meters wide, and five meters long?

5. A truck can haul 2 tons of rocks at a time. How many trips will it take to haul 5 tons?



Maximizing Math
Numbering with Nines
by Dave Youngs

In this month's *Maximizing Math* activity, students complete six sets of simple computations involving the number nine. As they do the problems in each set, students will discover obvious patterns they can use to complete the sets without having to perform all the computations.

This activity has three distinct mathematical aspects, each of which is valuable in its own rights. First, students get plenty of practice in basic arithmetic as they do the first few problems in each set. While students might be tempted to use a calculator, doing so would rob them of the opportunity to practice their computational skills in a problem-solving setting. Therefore, calculators should be discouraged.

Second, as students do the initial calculations in each set, they will discover patterns in the answers. The importance of patterns and pattern recognition in mathematics cannot be overstated. The mathematician and author Keith Devlin makes a strong case for this in his book, *Mathematics: The Science of Patterns,* which I recommend to you.

Third, students will be able to apply the patterns they discover to complete the problems in each set without having to do all the computations. This application of patterns is at the heart of inductive reasoning. It is also part of what Arthur Wiebe, one of the founders of AIMS, calls the mathematician's "pursuit of laziness." He notes that a good mathematician knows how to discover and apply patterns. This use of patterns to make his or her work easier (Wiebe's pursuit of laziness) is at the heart of mathematical thinking and reasoning.

While the mathematical aspects of this activity noted above are a key part of this activity, it goes beyond these three things and lets students actually play the role of mathematicians. They do this in the last part of the activity where they are challenged to come up with a set of computations of their own to explore and then to report their findings. These extensions might explore some other computational sets that use the number nine, since there are many more things that can be done with this fascinating number. For example, adding a two to each of the *nine numbers* (nine, ninety-nine, nine hundred ninety-nine, etc.) in the first set produces an even more interesting pattern than adding a one. Likewise, the fifth set can be modified by multiplying the nine numbers by something other than five. In fact, this set could turn into an interesting separate investigation by multiplying the nine numbers by all the numbers from one to nine and then comparing and contrasting the products in each of these sets. Another way to extend the activity is to use numbers other than nine in some of the sets. For example, in the second set, eights can be substituted for the nines with interesting results. It's important to note that numbers other than nine may not produce as many (or even any) recognizable patterns, but that this discovery is also important.

As students work on this activity, they should develop an increased appreciation for the power of patterns and their applications in mathematics. Be sure to leave lots of time for discussion at the end of this activity so that students can share some of their discoveries. This sharing time helps develop mathematical communication skills and is an invaluable part of the activity.

I hope you find this activity worthwhile. There'll be another one in the next issue. If you have any questions or comments, please feel free to contact me. My email is dyoungs@fresno.edu.

NUMBERING with NINES

In this activity you will work with various patterns stemming from operations with the number nine. In each set, start by doing the arithmetic and recording the answers until you have enough information to determine the patterns and extend them without doing any more computation.

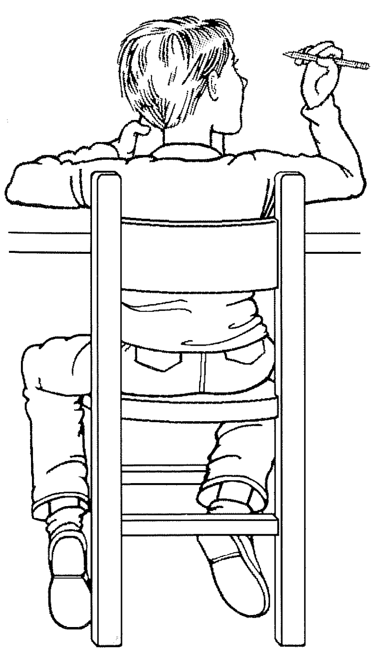
Here is your first set of problems:

$$\begin{array}{r} 9 \\ +\ 1 \\ \hline \end{array} \qquad \begin{array}{r} 99 \\ +\ 1 \\ \hline \end{array} \qquad \begin{array}{r} 999 \\ +\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 9999 \\ +\ \ \ 1 \\ \hline \end{array} \qquad \begin{array}{r} 99999 \\ +\ \ \ \ 1 \\ \hline \end{array} \qquad \begin{array}{r} 999999 \\ +\ \ \ \ \ 1 \\ \hline \end{array}$$

Describe the patterns you notice in the sums above. How did you use this pattern to find the remaining answers without doing the computation?

Here's another slightly different set of problems with another interesting set of sums. Again, use arithmetic to come up with enough sums to see the pattern, and then use the pattern to find the rest of the answers.



$$\begin{array}{r} 9 \\ +\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 99 \\ +\ 22 \\ \hline \end{array} \qquad \begin{array}{r} 999 \\ +\ 222 \\ \hline \end{array}$$

$$\begin{array}{r} 9999 \\ +\ 2222 \\ \hline \end{array} \qquad \begin{array}{r} 99999 \\ +\ 22222 \\ \hline \end{array} \qquad \begin{array}{r} 999999 \\ +\ 222222 \\ \hline \end{array}$$

Describe the pattern and how you used it to find the sums without doing all of the addition.

NUMBERING with NINES

This next set of problems is only slightly different from the previous one. See how quickly you can discover its pattern.

9 + 3	99 + 33	999 + 333
9999 + 3333	99999 + 33333	999999 + 333333

Describe the pattern and how you used it to find the sums without doing all of the addition.

The next set of problems involves subtraction:

9 - 4	99 - 44	999 - 444
9999 - 4444	99999 - 44444	999999 - 444444

Again, describe the patterns you found when doing these subtraction problems.

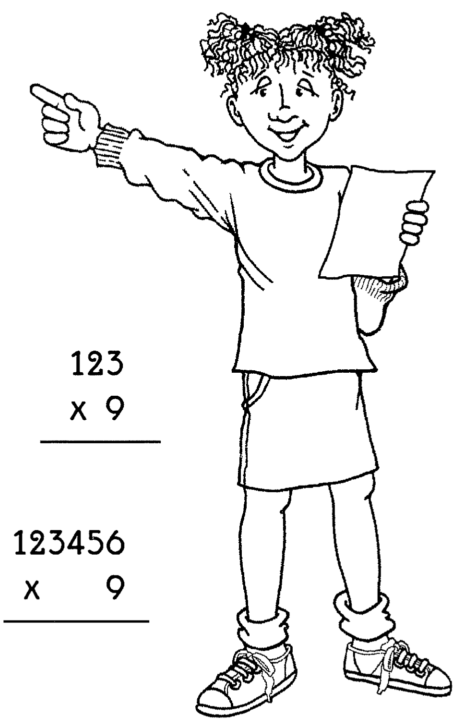
NUMBERING with NINES

Let's try some multiplication problems:

$$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array} \qquad \begin{array}{r} 99 \\ \times 5 \\ \hline \end{array} \qquad \begin{array}{r} 999 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 9999 \\ \times 5 \\ \hline \end{array} \qquad \begin{array}{r} 99999 \\ \times 5 \\ \hline \end{array} \qquad \begin{array}{r} 999999 \\ \times 5 \\ \hline \end{array}$$

What patterns did you discover when doing these problems?



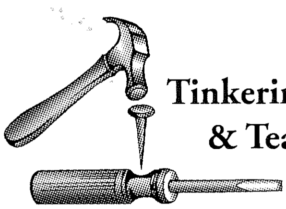
Now try these problems.

$$\begin{array}{r} 1 \\ \times 9 \\ \hline \end{array} \qquad \begin{array}{r} 12 \\ \times 9 \\ \hline \end{array} \qquad \begin{array}{r} 123 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1234 \\ \times 9 \\ \hline \end{array} \qquad \begin{array}{r} 12345 \\ \times 9 \\ \hline \end{array} \qquad \begin{array}{r} 123456 \\ \times 9 \\ \hline \end{array}$$

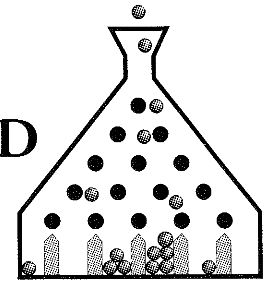
Again, explain how you extended the pattern to do this set.

Now that you have explored several different problems with nines and their patterns, try to make some problems of your own and explore those patterns. Use the back of this paper to show your work. Be prepared to share your discoveries with the class.



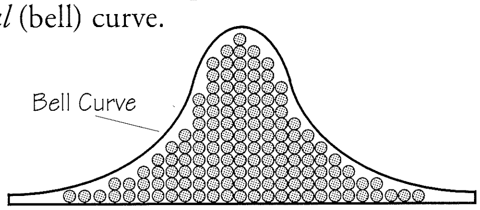
Tinkering, Toys, & Teaching

GALTON BOARD



by Jim Wilson

Sir Francis Galton (1822–1911) invented a device he called the *quincunx*. His quincunx was a glass-faced box containing several rows of pegs arranged in a triangular pattern. A funnel at the top of the box allowed steel shot (BBs) to be poured into the device. The BBs fell through the rows of pegs into compartments at the bottom of the box. The pegs deflected each falling BB to the left or right with equal probability. The distribution of BBs at the bottom of the quincunx approximated the *normal* (bell) curve.



Since the normal curve describes the distribution of so many things found in the real world, from the length of leaves on a tree to student test scores, Galton wanted a simple and visually interesting method for demonstrating the normal curve to those who attended his lectures. Quincunxes (often quite large and containing thousands of spheres) are commonly seen in science museums. There's a hypnotic fascination in watching the spheres fall through the device and, time after time, arrange themselves into a graceful bell-shaped curve at the bottom of the container.

I call the quincunx a Galton Board and I believe the device cleverly demonstrates several elementary probability and statistics concepts.

When I taught middle-school mathematics, I spent a lunch hour in the school shop and made a Galton Board by drilling holes in a piece of plywood and gluing short pegs into the holes. Marbles released into the device roll through, bouncing either to the left or to the right at each peg, until finally dropping into one of the bottom compartments. The device worked great (it's now semi-retired and hanging on my office wall), but most teachers don't have the materials or a wood shop to make a similar board.

After many unsuccessful attempts to tinker a Galton Board from common materials, I have finally succeeded in designing a board so easy to make that every teacher responsible for teaching probability and statistics concepts can make (or have their students make) several for classroom use.

After constructing your own Galton Board, you will use it to perform a simple probability experiment. I'll then describe how to perform two other seemingly different experiments. The data obtained from each experiment will then be compared to the data obtained from the other experiments. There's much to be observed and learned from these comparisons so gather the following materials and tools and let's get started.

Materials

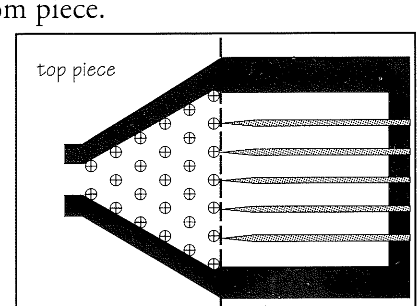
27 pushpins
Cardboard box
Marbles
Cup
5 pennies (or 5 nickels, 5 dimes, or 5 quarters)

Tools

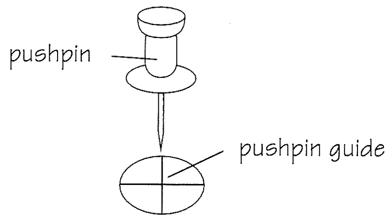
Scissors
Glue stick

Constructing A Galton Board

1. Make a copy of the *Pattern* pages. Cut along the dashed line at the bottom of both pieces.
2. Run a glue stick along the tab of the bottom piece where it's labeled "glue along this tab." Carefully align the top piece and glue it to the bottom piece.



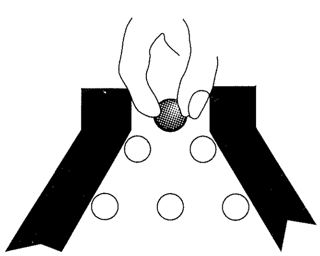
3. Run a glue stick over the back of the Galton Board pattern page and carefully glue the pattern, face up, to the inside of the box lid. The bottom edge of the pattern page should parallel the bottom edge of the box. The lid from a cardboard box (of appropriate size) works best since the sides keep the marbles under control. The lid from a copy paper box is ideal.
4. Cut a second piece of cardboard as wide as the pattern and 7 inches tall. Turn the box over and glue this piece to the bottom of the box lid *beneath* the triangular pattern. The purpose of this piece will be to cover the sharp points of the pushpins.
5. Using the crossed circles on the pattern page as guides, stick a pushpin into the cardboard at each location.



To keep construction simple, I purposely did not glue compartment walls at the bottom of the board. You may choose to do so.

Testing the Galton Board

Hold the Galton Board in one hand, lean it back at approximately a 45-degree angle, check that the bottom is level, and drop a marble through the top opening.



The marble should bounce repeatedly off the pushpins until it exits through one of the openings labeled *A* through *F*. If the marble consistently rolls to one side, check that you're keeping the bottom of the box level.

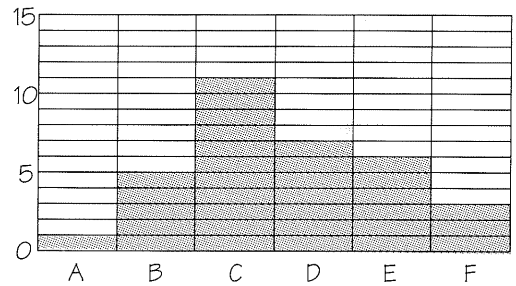
Using the Galton Board

Make a copy of the *Probability Experiments* page.

First Experiment: The Galton Board

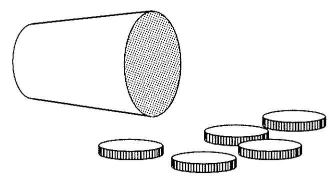
1. Set up and level the Galton Board.
2. Run a marble through the pins and observe which lettered section, A–F, it passed through. Record this outcome by shading the graph at the appropriate location. Repeat 31 more times for a total of 32 trials of the experiment.

The graph shows the results I obtained. Your graph will probably (no pun intended) be different.



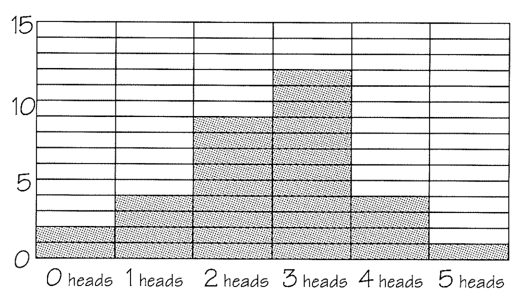
Second Experiment: Coins in a Cup

1. Put five pennies in a cup. Shake the cup and empty it onto your desktop.



2. Count the number of heads. Record this number in the appropriate column on the graph. Repeat this experiment 31 more times.

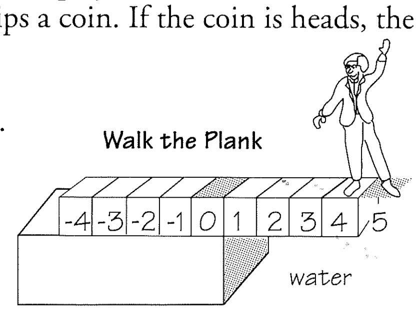
The graph I obtained is shown below.



Third Experiment: Walk the Plank

This experiment is a takeoff on the popular *Dunk Tank* in which a person sits on the end of a board over a tank of water. Players pay for the opportunity to throw baseballs at a target which, if hit, triggers a device that drops the person sitting on the plank into the water.

In *Walk the Plank*, a player starts at zero on a number line and flips a coin. If the coin is heads, the player moves one step to the right (positive direction). If tails, the player moves one step to the left (negative direction).



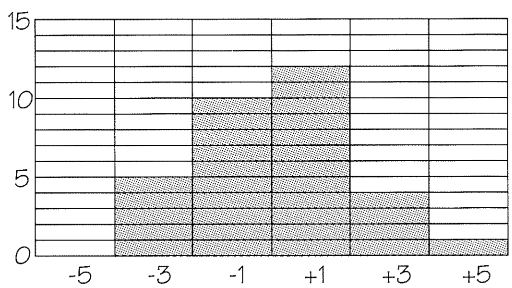
If the player is at position "4" on the fourth flip and flips a head, the player has to step off the end of the plank into the water.

1. To *Walk the Plank*, copy this number line on a sheet of paper. Place one of the coins from the previous experiment in the square labeled *0*.

-5	-4	-3	-2	-1	*0*	+1	+2	+3	+4	+5

2. Flip another coin and observe whether the coin is a head or tail.
3. If the outcome of the flip is a head, move the counter on the number line one square to the right, in the positive direction. If the outcome is a tail, move the counter one square to the left, in the negative direction.
4. Flip the coin exactly five times and record the location of the counter (after the fifth flip) on the graph.

   The graph shows the results of the 32 times I performed the experiment. [Note that in the 32 trials, the walker only fell into the water one time.]



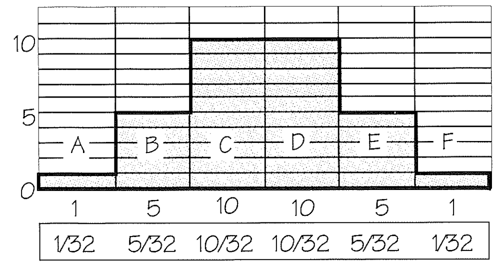
Comparing the Data

Now that you've completed all three experiments, let's examine and compare the data.

1. Although the graphs obtained by conducting 32 trials for each experiment are different in details, there is an obvious similarity about them; each graph has a hump in the middle and tapers off to either side.
2. The left end of the graphs represents those marbles that bounced "left" at every pushpin, five tails shaken from the cup, and flipping five tails in a row. The "humps" represent almost equal outcomes of left and right bounces or heads and tails. Everyday experience tells us that almost equal outcomes for each experiment are more likely than unequal outcomes. In short, the shapes of the graphs make sense.
3. In *Walk the Plank*, the positions -4, -2, 0, 2, and 4 cannot be reached after exactly five flips of the coin. The probability of reaching any of these positions after five flips is therefore zero.

The Theoretical Probabilities of Our Galton Board

There are exactly 32 different paths the marble can take through our Galton Board. The number of paths to each of the six possible compartments is shown below the horizontal axis of the graph. The fractions in the box are the theoretical probabilities. For example, the theoretical probability of reaching *A* is one-thirty second. Notice that the sum of all the probabilities equals one.



Exactly the same theoretical probabilities graph can be constructed for the other two experiments. (Note: If you would like a detailed explanation of how the theoretical probabilities are computed, you can email me at jawilson@fresno.edu)

The graphs of the three experiments are similar because the three experiments are essentially the *same* experiment. The marble must bounce to the left *five* times in a row to reach compartment *A*. The left-most column of the *Coins in a Cup* graph can only be filled in if *five* tails are emptied from the cup. The only way to reach the -5 position on the number line is to flip *five* tails in a row.

Comparing Arithmetic Concepts to Probability Concepts

Little attention is given to the fact that the processes and procedures we teach when introducing students to elementary probability concepts are quite different from those we teach in elementary arithmetic.

To clarify this point, think of combining two apples with three apples as a simple experiment in arithmetic. Every time you perform this experiment, you count "five apples" as the outcome. Additional repetitions soon convince you that you will *always* count five apples when you combine two apples with three apples.

Now consider an elementary probability question typical of those that appear in textbooks. A brown paper bag contains five spheres, all the same size, weight, and texture. Three of the spheres are colored green, and the remaining two are red. A typical "probability" question asks, "What's the probability of drawing a red sphere from the bag?" The textbook answer is two-fifths. This is the "theoretical" answer to the question.

Instead of an abstract textbook exercise, let's look at the same problem from a different perspective. Color three table tennis balls green and two red. Put them in a paper bag. Draw one ball from the bag. The outcome of doing this experiment is either a red or green table tennis ball, not a ratio. For example, I reach into the bag and draw out a red ball. I reach in a second time and withdraw a green ball! Whatever result I get by doing the experiment once doesn't tell me the color of the ball I will next draw. But, if I repeat the experiment a large number of times and record the outcome of each experiment, I will find that I drew a red ball approximately 40% (two-fifths) of the time. This is the "experimental" answer to the question.

The fact that there are two "probabilities" raises the important pedagogical question as to whether students should experience a balanced approach to learning probability concepts or simply be taught a few counting techniques for computing theoretical probabilities.

Doing these experiments with students should help you answer the question for yourself.

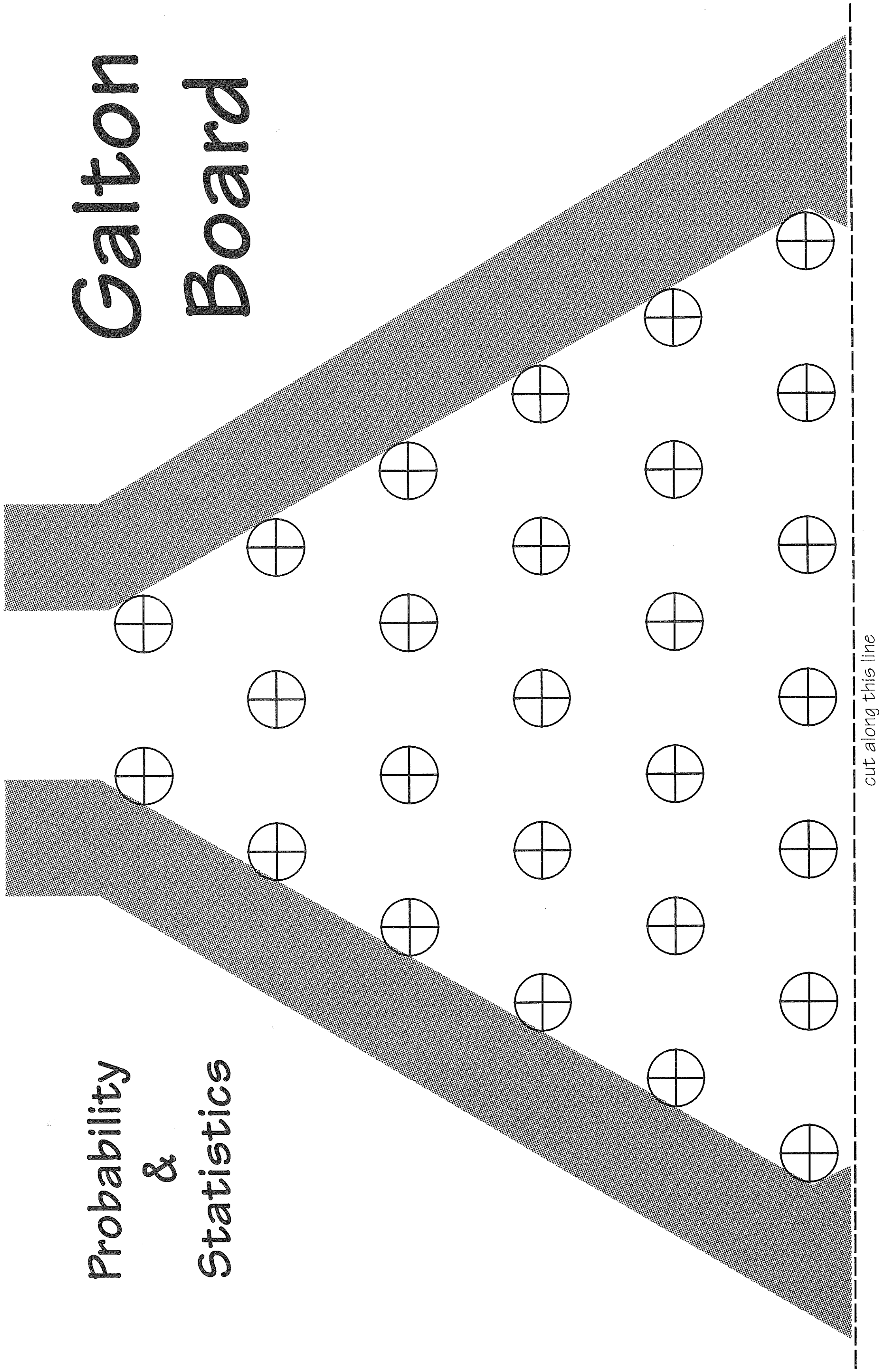
Technology

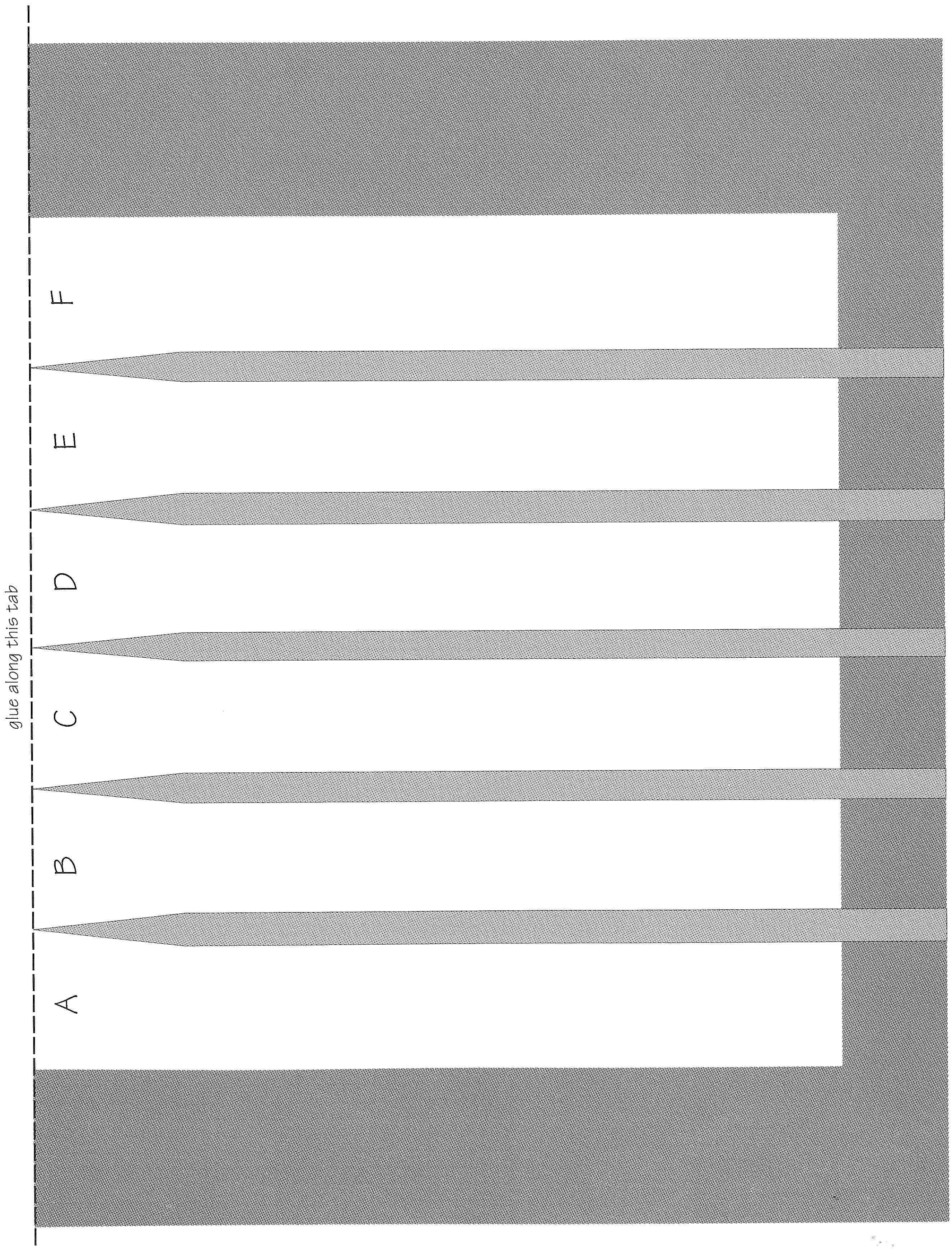
Teachers with access to Texas Instruments programmable graphing calculators can have students program them to quickly generate data for *Walk the Plank*. Enter the following program into one calculator and then use the link cable to quickly copy the program into the other calculators. Program comments appear in brackets and are not part of the program.

```
PROGRAM:WALK
ClrHome                [clear display screen]
0->C                   [set walker at "0"]
For (X, 1, 5)          [flip coin 5 times]
int(rand2+1)->F        [generate "1" or "2"]
If F = 2
Then
C+1>C                  [move one step left]
Else
C–1->C                 [move one step right]
End
End
Disp C                 [display final location]
```

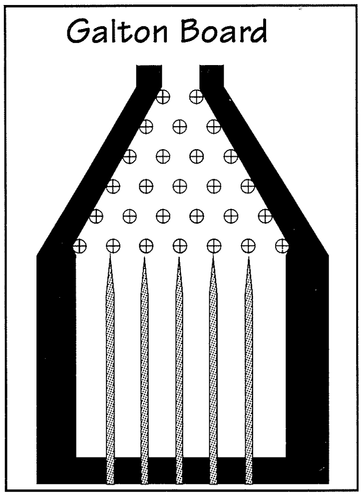
The program flips a coin five times and reports the final destination, on the number line, of the plank walker.

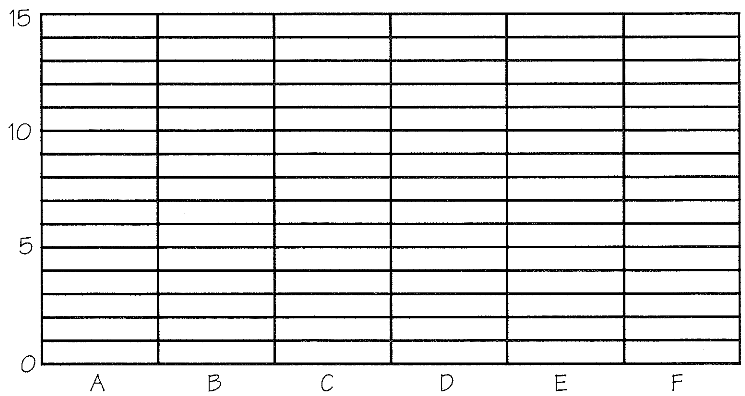
In the next *Tinkering* column I'll report on the outcome of my observing and counting the growth of a single-cell organism in 20 mL of water containing a single grain of uncooked rice.

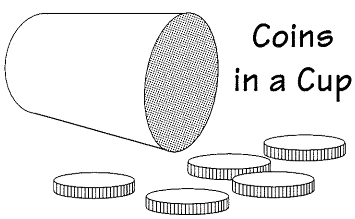

Galton Board
Probability & Statistics
cut along this line

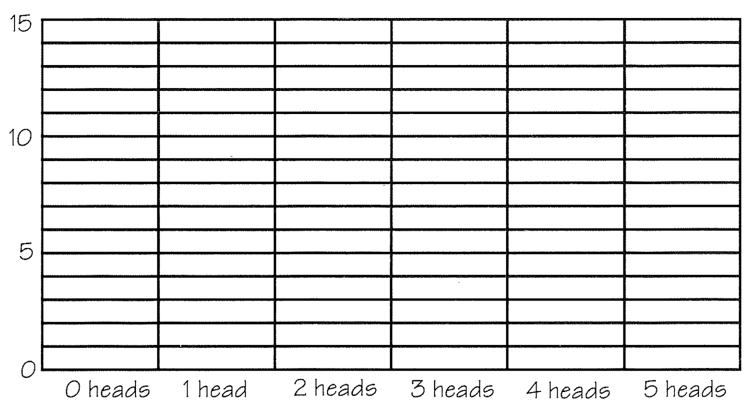

glue along this tab
A
B
C
D
E
F

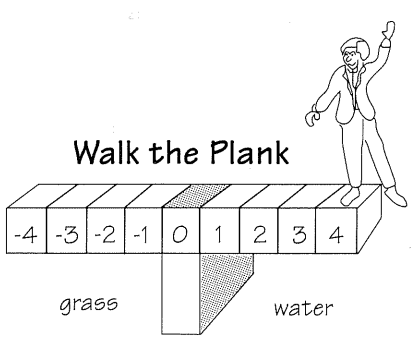
Probability Experiments

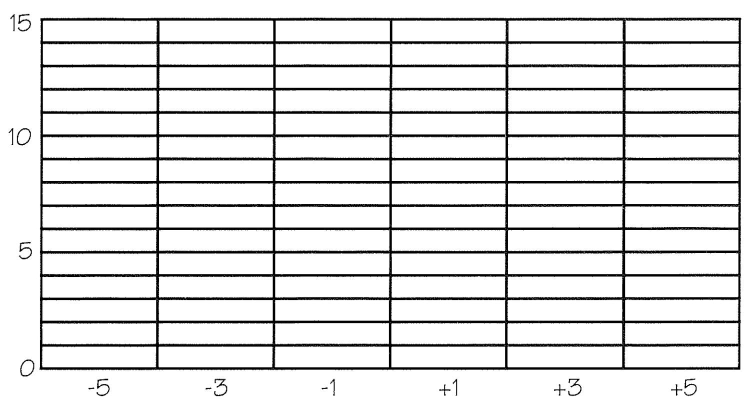










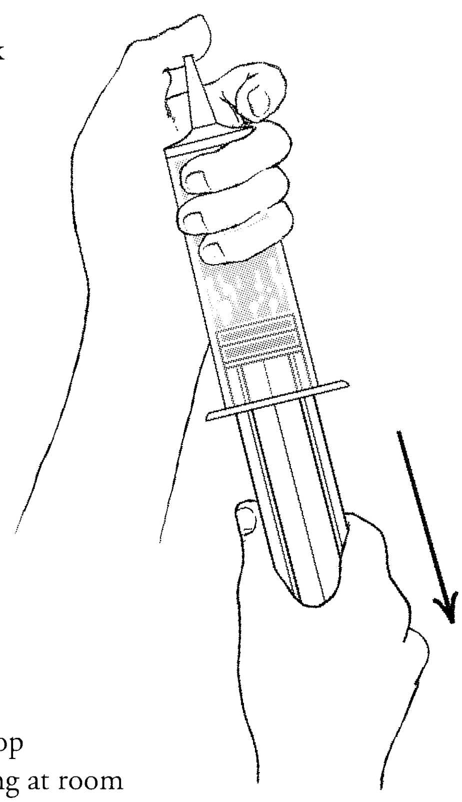


BOILERMAKERS

by Dave Youngs

Background Information

Boiling is a change of phase from liquid to gas that occurs beneath the surface of a liquid. (When this change of phase occurs at the surface, it's called evaporation.) Under normal atmospheric pressure, water boils at 100 degrees Celsius. At this temperature, the kinetic energy of many of the water molecules is great enough that these molecules break free from other water molecules and form water vapor (a gas) beneath the surface of the water. The vapor pressure of these molecules is enough to overcome the pressure of the surrounding water creating bubbles of water vapor which rise to the top in the process we call boiling. Since water pressure is directly tied to the surrounding pressure pushing down on the surface of the water, changing this outside pressure changes the temperature at which water boils. For example, increasing this outside pressure by using a pressure cooker causes the boiling point of water to rise significantly. Conversely, when this pressure is reduced, the boiling point of water is lowered. (This is why eggs must be boiled longer at higher altitudes where the atmospheric pressure is less.) If the outside pressure is reduced enough, as this activity demonstrates, water can boil at room temperature.



In this activity an oral syringe is used to diminish the pressure surrounding a given volume of water, allowing it to boil. When the syringe is filled with water and the opening sealed off, pulling down on the plunger greatly reduces the pressure on the water. This reduction in pressure is enough to allow water vapor bubbles to form and rise to the top where a partial vacuum is created. The water in the syringe really is boiling at room temperature! Releasing the plunger increases the pressure again and the water vapor returns to its liquid form.

Materials

Oral syringe
Plastic or rubber cap for syringe (optional)
Water

Procedure

1. Fill the syringe with water until it is about half full.
2. Hold the syringe vertically with the opening up and push the plunger up until all the air is removed.
3. Cap the syringe with an airtight cap, if available—otherwise place your thumb so that it tightly covers the opening.
4. Hold the syringe securely with one hand and pull down on the plunger with the other.
5. Observe what happens. Release the plunger and repeat this process.
6. Discuss your observations.

Table of Contents
February 2002, Volume XVI, Number 7

P = Primary Grade Activities M = Middle Grade Activities U = Upper Grade Activities

Web Page: http://www.aimsedu.org/

Activities Integrating Math, Science, & Technology
AIMS

AIMS Education Foundation
P. O. Box 8120
Fresno, California 93747-8120